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AN EXPERIMENTAL STUDY OF THE BEST MINIMUM WAVE-LENGTH FOR VISUAL ACHROMATISM

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ABSTRACT. A special telescope is described, in which the chromatic aberration can be varied continuously by means of a zero-power lens sliding within the telescope tube, the conditions being such that this movement does not introduce any other undesirable aberrations. This apparatus has been constructed, and by its aid many determinations have been made as to the best type of achromatism for visual observations, in daylight, in artificial light, and on astronomical objects. Several observers were employed to make settings, and their opinions as to the most desirable type of achromatism are tabulated and compared. In general it is found that a moderate amount of undercorrection is needed to give a truly colour-free image, that less undercorrection is required if the best definition is to be obtained, and that a slight overcorrection is desirable for astronomical work.

INTRODUCTION

IT has long been the custom, when designing a small telescope or microscope objective, to trace the *C* and *F* hydrogen rays through the system, and to choose suitable radii and glasses to make these rays meet at the image formed by the objective. This procedure, as is well known, does not ensure that light of other wave-lengths will meet at that focus also; in fact they will not do so, but the spectrum will be doubled back on itself, the minimum focus occurring at a wave-length of about 555*. The extreme ends of the spectrum are both badly out of focus compared with the minimum; but opticians justify this by pointing out that to a normal eye the peak of the luminosity curve of sunlight in the prismatic

* Wave-lengths will be expressed here in terms of ·001 μ, or 10 Ångström units.

spectrum is at this wave-length 555, so that the red and blue ends of the secondary spectrum arising at the image of an achromatic objective will be so faint as to be insensible. While this "explanation" is adequate for small telescopes working at low powers, it entirely fails for a large instrument, because there the departure from perfect focus of the extreme violet end of the spectrum is far too great to be ignored. In this case, as every astronomer knows, the best image of a star is a yellowish-white dot surrounded by a vast pale-violet halo. In low-power telescopes there is still some halo, but it is so small in size as to be frequently overlooked.

The achromatism of an objective is, however, modified by the use of an ordinary unachromatic eyepiece. At high powers the pencil of light through the eyepiece is very thin, and the image is therefore largely unaffected by any lack of achromatism there; but at lower powers, especially in night glasses and binoculars, the emerging pencil may have a diameter of 5 to 8 mm., in which case the lack of achromatism in the eyepiece has a considerable effect on the final image seen. By means of a test such as the Vogel test to be described below, the actual minimum focus for the telescope as a whole can be determined and compared with the calculated minimum of the objective alone, when it will be found that the wave-length of the minimum focal length has become very materially lowered by the eyepiece. On testing an ordinary 6× prismatic binocular in this way, the final minimum wave-length was found to be in the violet, at about 420. This is probably rather extreme, but in two "Watkin Position Finders" which were also tested the ultimate minima were at about 470 and 520 respectively. A modern pocket three-draw terrestrial telescope of French origin had its minimum at about 420, while that of an old three-draw telescope with a wooden body was at about 520.

For large astronomical objectives, on the other hand, the long experience of many astronomers has shown the desirability of raising the minimum to almost 570 in the objective, and for the high powers generally employed in astronomical work this means about 570 in the ultimate image also. The actual figures for the Lick and Yerkes objectives, for example, are:

Objective	Aperture in.	Focal length ft.	Minimum wave-length
Lick*	36	57	565
Yerkes†	40	63	568

It is hopeless to try to improve on these values of the minimum as they are based on a mass of experience and practical trial in many parts of the world and by many different observers; but for small astronomical, and more particularly for terrestrial, telescopes there is still a very grave uncertainty as to the best minimum to aim at in achromatism.

In order that systematic determinations of this desirable minimum might be made by various observers on various objects, a 3-inch telescope was constructed, inside which was a sliding zero-power lens with considerable chromatic aberration. By moving this lens, the state of achromatism of the image could be continuously

* *Pubs. Astron. Soc. of Pacific*, **2** 1890 (258).
† *Astrophys. Journ.* **27** 1908 (249).

altered at will*. The process of the design, and the results obtained by its aid, will now be described.

The Chromatic Telescope

The apparatus for this investigation consisted of a telescope objective of 7·6 cm. aperture and 50 cm. focal length, chromatically overcorrected, together with another sliding lens having no power, but strongly undercorrected chromatically. When this second lens was placed in the plane of the image formed by the objective, it would obviously produce no effect whatever on the image, but as it was moved towards the objective, its effective aperture would become gradually increased owing to the widening of the cone, causing it to introduce increasing amounts of chromatic undercorrection. In this simple way the final image could be given any desired state of achromatism within the entire useful range of the visible spectrum.

Until recent times it has been usual to express a given condition of achromatism by stating two wave-lengths which are combined at the image, but in this work it has been found most convenient to state simply the wave-length of the minimum focal length. For example, if a lens has been achromatised by bringing the hydrogen *C* and *F* rays together at a common image-point, this implies a minimum focal length for the yellow-green light of wave-length $\lambda = 555$; hence a lens achromatised in this way would be said to have a "minimum focal length wave-length of 555" or, more briefly, a "minimum wave-length of 555."

The objective used in this work was designed to produce a minimum focal length for the red hydrogen *C* line ($\lambda = 656$), and the zero-power compensating lens was so constructed that at its nearest distance of 30 cm. from the objective the minimum focal length became shifted to the blue hydrogen *F* line ($\lambda = 486$). Hence, by simply moving the compensator over a range of 20 cm. from the eyepiece towards the objective, a complete continuous range of minimum wave-lengths could be obtained from $\lambda = 486$ to $\lambda = 656$. It was thought that this range would include all possible choices of best achromatism by all observers, but actually it would have been better if the range had extended down to 430 instead of 486.

As regards calibration, the minimum wave-length at four compensator positions was calculated for the primary image by ray-tracing, but the complete instrument including an eyepiece was ultimately calibrated experimentally by means of the Vogel test. This calibration depended on the eyepiece used, on the aperture, and on the distance of the object, making it necessary to determine a series of calibration curves and to use the appropriate one for each series of observations.

Various test-objects were used, some natural but mostly artificial, each observer being asked to move the compensator until the best-focussed image was either (1) as free from colour and coloured fringes as possible, or (2) as sharply defined as possible. A range of such settings was of course taken and the average found. The conclusions deduced from the various tests are given later in Tables I, II, and III.

* The first to use such a "chromatic plano-parallel plate" appears to have been Dr Hugo Schroeder, but he never performed any really systematic tests with it. See *Centr. Ztg. f. Opt. u. Mech.* **20** 1899 (71, 81).

To give an idea of the relative seriousness of the secondary spectrum in this telescope, it is interesting to note that for a normal thin achromatic lens with the usual *C* and *F* achromatism, the *C*–*F* rays will be out of focus by twice the Rayleigh limit and the 555 ray by half the Rayleigh limit in the opposite direction, if the focal length is given by $f = 2A^2$, where A is the aperture (in cm.). This may be regarded as a convenient "tolerance," because the *C*–*F* image will still approximate to a point, although only 40 per cent. of the light will appear in that image, the rest being scattered into the halo.

The chromatic telescope used in this work had a useful working aperture of 6·4 cm., and would therefore satisfy this tolerance if its focal length had been 80 cm. Actually the focal length was only 50 cm.; therefore this telescope may be said to have about $1\frac{1}{2}$ times the tolerance of the secondary spectrum. Now an ordinary *F*/15 astronomical telescope of 3 inches aperture is just about at this limit; we conclude, therefore, that our chromatic telescope has a secondary spectrum comparable with that of a normal $4\frac{1}{2}$-inch astronomical telescope.

By comparing the net thickness of crown and flint in a microscope objective with that of telescope objectives, it is found that the most usual achromatic objectives (i.e. $\frac{1}{8}$ and $\frac{1}{12}$ inch) have the same seriousness of the secondary spectrum as a 4- or 5-inch astronomical telescope, in terms of the limit defined above. Hence this "chromatic telescope" is representative both of the usual amateur sizes of astronomical telescopes and also of the most widely used microscopes. It is more severe than the usual theodolite or level telescopes and low-power microscope objectives. The permissible ranges of minimum focal length colour found in these experiments will therefore be safe in all cases except large astronomical telescopes for which extensive experience is already available.

The Optical Design of the Telescope Objective

Whenever experiments have been performed hitherto with the aim of ascertaining the effect of chromatic aberration on an image, no account has been taken of the presence of spherical aberration, one image being simply mentally compared with another having a different colour distribution. When an observer is trying to decide which of two images is the better from the point of view of colour, it is very important that his judgment should not be upset by the presence of varying unknown amounts of spherical or other aberration. Hence the optical system used for these experiments was designed so that at all positions of the compensator the spherical aberration should never exceed $\frac{1}{8}$ of the Rayleigh quarter-wave limit.

As regards the objective, therefore, the flint-in-front type was chosen as being more likely to be free from undercorrected zonal aberration than the ordinary type, and it was made to fulfil the sine-condition by introducing a small air-space between the two components. Actually this air-space "assisted" the zonal correction to such an extent that it ultimately reached half the tolerance in the overcorrected direction! Probably the best results would have been obtained with a separated crown-in-front lens; or, if a slight difference in curvature at the air-space

were included, it would probably have been possible to remove the zonal aberration altogether. Owing, however, to considerations of economy, the choice of radii was greatly restricted by the necessity of using only the manufacturers' stock tools. In the end it was found possible to use existing tools for all the surfaces of both lenses except the last surface of the objective, for which a tool was made up specially.

The lens was achromatised by the $(d-D)$ method* for a minimum focus at the hydrogen C line ($\lambda = 656{\cdot}3$), the marginal spherical aberration being corrected for that wave-length. Fulfilment of the sine condition was secured to within half the tolerance, and the entire absence of any coma in the final image confirmed the adequacy of this correction.

The process of design was briefly as follows. Two plates of suitable hard crown and dense flint glass were obtained from Messrs Chance, and small prisms were cut from them for accurate measurement of refractive index and dispersion. These measurements were performed on a Société Genevoise spectrometer, reading to single seconds of arc, with the following results:

	Flint	Crown
N_C	1·61704	1·51380
N_F	1·63423	1·52235
$N_{G'}$	1·64466	1·52717

Using Prof. Conrady's interpolation formula

$$N = N_0 + \frac{a}{\lambda} + \frac{b}{\lambda^{\frac{7}{2}}},$$

the constants N_0, a, and b of each glass were calculated, and the dispersions δN determined in the neighbourhood of the C line, for a range of colours corresponding to the usual value $(N_F - N_C)$. The computing figures for achromatism in the neighbourhood of the C line, therefore, were:

	Flint	Crown
N	1·61704	1·51380
δN	·01674	·00885
ν	36·86	58·06

whence the total curvatures of the crown and flint components for a lens of 50 cm. focal length were determined.

By means of an approximate thin-lens formula, that curvature of the first surface was calculated which would give freedom from coma; and after assigning suitable thicknesses, a marginal and paraxial ray were traced through this lens, the last radius being determined by the $(d-D)$ method to give achromatism in the neighbourhood of the C line. A considerable amount of negative (overcorrected) spherical aberration was found, which was removed by introducing a suitable air-space. The general effects of air-space on the properties of the lens were then ascertained by tracing through several trial separations (the last radius of each

* A. E. Conrady, *Monthly Notices, R.A.S.* **64** 1904 (182); or *Dictionary of Applied Physics*, vol. **4**, p. 224; H. W. Lee, *Trans. Opt. Soc.* **22** 1920–21 (227).

trial being determined for strict achromatism) and plotting the results on a graph (Fig. 1).

As a separation of the components greatly increased the "offence against the sine condition" (O.S.C.), it was necessary to allow a certain amount of negative O.S.C. in the thin-lens design which would disappear on breaking the contact. It was found from this graph that a separation of ·256 cm. would give complete marginal spherical correction, and this separation was therefore adopted in all the subsequent calculations and in the final lens.

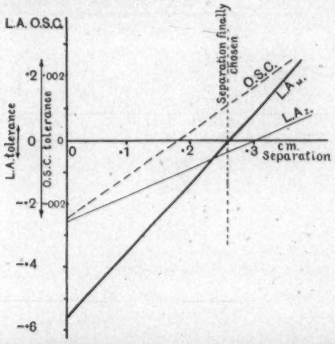


Fig. 1. The Effects of separating the Components of the Objective

The longitudinal aberration of the ·7 zone here amounted to − ·0371 cm., the tolerance being about ·06 cm. The negative sign indicates the unusual zonal overcorrection. The final specification of the objective, in centimetre measure, was:

$$r_1 = 21{\cdot}346$$
$$d_1 = {\cdot}76 \text{ (flint)}$$
$$r_2 = 9{\cdot}307$$
$$d_2 = {\cdot}256 \text{ (air)}$$
$$r_3 = 9{\cdot}307$$
$$d_3 = 1{\cdot}14 \text{ (crown)}$$
$$r_4 = -222{\cdot}3$$

Focal length = 48·19
Full aperture = 8·0

The Optical Design of the Compensator

On considering the design of the compensator, it was evident that, whatever shape the lens might have, it would produce no effect whatever on the image when it is in, or practically in, the plane of that image. Hence the problem was to design a lens which should have infinite focal length, and should have such a shape as to correct the spherical and chromatic aberrations of the objective for hydrogen *F* light, when at a distance of about 30 cm. from the objective. This would then automatically leave unchanged the original correction for *C* light when the compensator is moved into the image-plane.

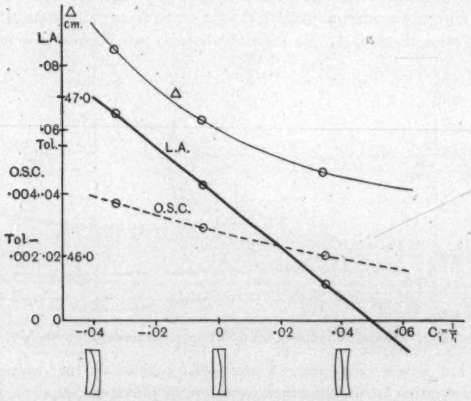


Fig. 2. Effects of bending the Compensator (*F* light; $d = 27{\cdot}43$ cm.)

The $(d - D)\,dN$ sum of the objective was first determined for *F* light, and an analytical solution made for a lens which would yield an equal and opposite value of the chromatic sum with infinite focal length, when at a distance of 30 cm. from the objective. This enabled the total curvatures of crown and flint to be calculated; and from a series of three bendings, a graph was drawn by which that bending could be chosen which would give spherical correction in the final image for *F* light (Fig. 2). For each bending the last radius was determined to make the total $(d - D)\,dN$ sum for the whole system in *F* light equal to zero. Small corrections had to be made, however, because the finite aperture and thickness of the lens imparted to it a slight resultant positive focal power, necessitating a strengthening of the concave component; and also because of the desirability of employing stock

tools where possible. The design led to the form of a convex flint lens in front of a concave crown, with the specification (in cm.):

$$r_1 = 16{\cdot}50$$
$$d_1 = {\cdot}6 \text{ (flint)}$$
$$r_2 = -42{\cdot}39$$
$$d_2 = {\cdot}4 \text{ (crown)}$$
$$r_3 = 11{\cdot}77$$
$$\text{Full diameter} = 3{\cdot}8$$

This compensator is such that, when placed at 26·4 cm. from the objective, it gives a minimum focal point for *F* light, free from spherical aberration, at a distance Δ from the objective; and also, when moved to a distance 44·1 cm. from the objective, it gives a minimum focus in *C* light, again free from spherical aberration, and at the same distance Δ. No account whatever was taken of the intermediate

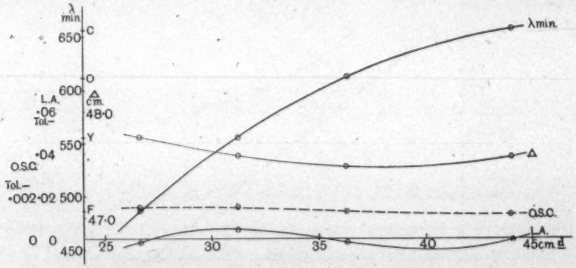


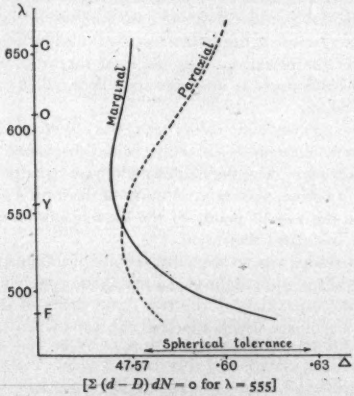
Fig. 3. Theoretical Properties of the Chromatic Telescope (no eyepiece)

positions, but it was found that at any given position of the compensator the spherical aberration for the minimum wave-length corresponding to that particular state of adjustment was always very small, approximately one-sixth of the Rayleigh limit at the best focus. The variation in spherical aberration and coma as the compensator is moved is clearly shown in Fig. 3.

The method of determining the theoretical calibration curves of the optical system was briefly as follows. Using the interpolation formula $N = N_0 + \frac{a}{\lambda} + \frac{b}{\lambda^4}$, the refractive indices for both glasses were calculated for the wave-length $\lambda = 555$ ("yellow"), and for $\lambda = 612$ ("orange"). For each wave-length, marginal and paraxial rays were traced through the objective, and the $(d - D)\,dN$ sum determined. The compensator was placed at a likely position and the rays continued through it to the final image-point. In general there would still be a residual $(d - D)$ sum outstanding, due to the guessed position of the compensator being incorrect; so three such trials were made, enabling graphs to be drawn from which

the correct separation to give a minimum focus at that wave-length could be read off. The accompanying changes in Δ, L.A., and O.S.C. were also shown on these curves.

The complete behaviour of the system was then shown on a graph (Fig. 3) connecting d, the separation of the objective and compensator, with the corresponding minimum wave-length, spherical aberration (L.A.), offence against the sine condition (O.S.C.), and the image-distance from the back surface of the objective (Δ). From this graph it is seen that the marginal spherical aberration never exceeds $\frac{1}{8}$ of the Rayleigh limit of ·055 cm., and the O.S.C. is about half its tolerance of ·0025. The position of the image unfortunately varies a little, but this appears to be unavoidable in the design. In using the instrument, therefore, it is necessary



[$\Sigma (d - D)\, dN = 0$ for $\lambda = 555$]

Fig. 4. Marginal and Paraxial Colour-curves for the Complete System ($d = 31{\cdot}2$ cm.)

to refocus the eyepiece at each setting, but this is no real hardship, as it is always advisable to observe the inside and outside focus effects at any state of adjustment of the apparatus.

The colour-curves of the telescope, when adjusted to give a minimum focus at 555 (i.e. when $d = 31{\cdot}2$), are shown in Fig. 4. The horizontal distance between these two curves at any point is the marginal spherical aberration for that wave-length. It is interesting to note that the $(d - D)$ method secures the desired minimum focus *for the ·7 zonal rays*.

The Mounting for the Lenses

The lenses were made by Messrs Wray, Ltd., of Bromley, Kent. The two components of the objective were fitted into a special mounting in which their separation could be varied as desired by the use of three push-pull screws. These were also valuable in squaring-on the two components relative to each other. This objective cell was screwed to one end of a stout brass tube carrying an eyepiece fitting at the other end. The compensator was mounted in a second short tube sliding easily within the main tube; and a knob fixed to the inner tube projected through a slot cut along the main tube, to enable the observer to adjust the compensator position as desired. The separation d between objective and compensator was read directly from a celluloid centimetre scale fixed alongside the slot, whence, by application of the calibration curve, the minimum wave-length could be ascertained at once. The aperture of the system was controlled by a thin card diaphragm attached in front of the objective. The inside of the tubes was lined as far as possible with dead-black paper to minimise stray light. This mounting was made in the College workshop.

Before commencing systematic colour-tests, the objective was submitted to an axial Hartmann test to determine the actually realised aberration. A diaphragm with holes on thirteen zones was used, the illuminant being a red artificial star; panchromatic plates were, of course, necessary. A curve of the form $a = A + BY^2 + CY^4$ was drawn through the various points by the least-square method, and it agreed fairly well with the calculated aberration.

As the zonal aberration was so large that the use of full aperture led to a slight deterioration of the image, most of the colour observations were made at an aperture of 6·4 cm. ($F/7$), the separation of the two components of the objective being adjusted to give a star image which appeared the same when expanded on either side of the focus.

Calibration of the Scale

The theoretical curve (Fig. 3) applies to the objective and compensator alone, assuming an object at infinity. In practice, however, this curve was of little value, as it was necessary to use uncorrected eyepieces and a relatively near object. As a distance of about 20 m. was the largest available in the open air for the colour-tests, a Vogel test was performed for that object-distance, with each of the eyepieces and apertures required, as follows.

At one end of a long passage an artificial star was erected; this consisted of a pinhole on which white light from an arc lamp and mercury light were focussed simultaneously by a 1-inch microscope objective. Vogel's* test is, briefly, that if the image of a star is viewed with an eyepiece placed just *within* the focus of the lens under test, a diffuse coloured disc is seen. But if this is now viewed through a 60° prism placed close behind the eyepiece, the various colours are spread out

* H. C. Vogel, *Monatsber. d. Akad. Berlin*, 1880 (433).

into a spectrum which will vary in width, being narrowest at the wave-length of minimum focus (Fig. 5). If the eyepiece is set exactly on the focus for any one wave-length, the spectrum will contract to a point at that wave-length. Further, if the eyepiece is withdrawn a little *beyond* the image, the spectrum will appear cigar-shaped, the widest part being now at the minimum wave-length.

With the composite (white plus mercury) star that was used for these calibration tests, the white light was spread out into a spectrum which appeared to have the bright yellow, green, and blue mercury lines impressed on it. In this way the compensator position corresponding to a minimum focal length at each of the mercury lines in turn could be quickly determined by a few trial settings. To make the calibration curve more complete, the mercury lamp was extinguished and salt was fed into the arc. This gave a continuous spectrum with the sodium line strongly marked, and the compensator was moved until the waist in the spectrum coincided with this line.

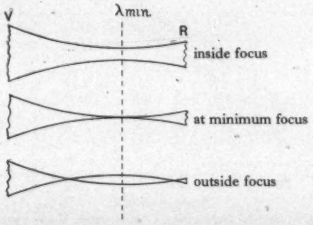


Fig. 5. Shape of Spectrum seen through an Eyepiece Prism

These known points did not of course fall on the theoretical calibration curve of the compensator, but up to 2 cm. from it, the curve joining the experimental points being, however, roughly parallel with it. The eyepieces used in this work were a Zeiss 7 mm. Huygenian and a Zeiss 15 mm. Ramsden eyepiece, giving magnifying powers respectively of 70 and 33. The calibration curves are given in Fig. 6, and show at once how much these uncorrected eyepieces really do lower the minimum wave-length.

The Test Objects

In order to ascertain the best minimum wave-length for terrestrial telescopes, theodolites, etc., it was decided to set up outdoors such objects as black letters on a white ground, white letters on a black ground, a contrasty and a dull half-tone photograph, and finally various coloured pictures and natural objects. The observer was in each case asked to set the compensator so that the image appeared either most sharply defined or most free from colour. There was always a considerable difference between these two settings, showing that, in general, sharp definition and freedom from colour-haze are incompatible.

A number of pictures were then observed in the light of a gas-filled lamp, a convenient arrangement being to place a suitable lantern slide in front of a "Fullolite" lamp. The figures obtained for this light should be of chief value to designers of microscopes.

Finally, a number of observations were made on the Moon and other astronomical objects; some at a power of 28 and some at 50 to the inch aperture. These results should interest makers of small astronomical telescopes, but they cannot be regarded as giving information as to the performance of a really large instrument. In order that objects at an infinite distance might be used, it was necessary to calibrate the instrument for that object-distance.

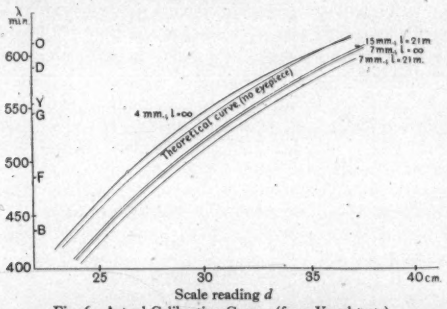


Fig. 6. Actual Calibration Curves (from Vogel tests)

Least-colour Tests in Daylight

In these series of settings, the observer was in each case asked to set the compensator so that the best-focussed image was as free from colour as possible. A considerable number of such settings were made by various observers, under as many different conditions as could be conveniently explored, with the results shown in Table I.

It should be noted that both bright and dull daylight were employed, using either a contrasty black-and-white object or a dull diffuse one, and also that two eyepiece powers were used. It was noticed during these tests that sharp clear objects in bright sunlight are extraordinarily sensitive, and that a slight increase in the minimum wave-length immediately introduces a purple halo, while a slight decrease produces a greenish or reddish halo according to the focussing adjustment. As the light becomes dull or the magnification is reduced, with diffuse objects, the range of uncertainty and the difficulty of making precise settings increases very considerably. In dull light, settings become extremely uncertain because the coloured halos are really too faint to be appreciable.

Experience in the use of the apparatus always makes for quicker and more definite settings. A beginner will often hardly see any difference in the image over

the entire range, yet, after he has done three or four settings, his range will have reduced to 1 or 2 cm. only. A certain amount of bias arises in the settings, due to memory both of the appearance of the image and also of the position of the hand on the knob which controls the compensator. It would have been better if this lens had a rack and pinion movement so that the observer would not be able to remember his previous settings.

Those observers who are familiar with ordinary astronomical telescopes have a tendency to *expect* the purple halo usually to be seen with such telescopes, and are therefore liable to give a high setting with this apparatus.

It is strange how some observers just glance into the eyepiece and set the compensator at once, whereas others often take five or six minutes to make a setting. The more rapid method is rather to be preferred, as it minimises colour-fatigue in the eye.

TABLE I. *Settings for Least Colour in the Image*

Type of object	High contrast						Low contrast					
Eyepiece power per inch	13			28			13			28		
	Obs.	Av.	No. of Obs.	Obs.	Av.	No. of Obs.	Obs.	Av.	No. of Obs.	Obs.	Av.	No. of Obs.
Sunlight	*A*	512	12	*A*	504	8	*A*	502	13	*A*	507	9
	B	524	9	*B*	523	5	*B*	526	4	*B*	517	4
	C	532	3	*E*	530	1	*J*	500	9	*J*	478	5
	E	542	1									
	F	520	7									
Highest individual setting		542			531			546			532	
Average setting		**520**	32		**512**	14		**505**	26		**490**	18
Lowest individual setting		491			486			456			469	
Shade	*A*	515	21	*A*	505	14	*A*	497	15	*A*	498	9
	C	496	2	*C*	537	3	*B*	519	2	*B*	547	2
	D	512	7	*G*	512	6	*G*	485	4	*C*	539	2
	E	522	6	*H*	537	4				*H*	530	6
	K	516	6									
	L	516	5									
Highest individual setting		530			549			552			569	
Average setting		**515**	47		**515**	27		**497**	21		**517**	19
Lowest individual setting		491			493			472			476	

A few settings were made on coloured pictures and on the leaves of trees, but the change in the apparent colour of the image due to a shift of the minimum wave-length was so slight as to be unnoticeable to the eye over a very large range indeed, frequently no difference being observed over the whole range of the instrument. It was decided, therefore, to confine deductions to black-and-white objects only.

As a result of many and various tests on different objects by different observers, the conclusion is reached that, to secure an image which has the greatest possible freedom from colour-haze, the ultimate minimum wave-length should be between 490 and 520.

These figures, in practice, mainly apply to the achromatising of prismatic binoculars, rifle sights, and pocket terrestrial telescopes. When planning such a telescope, the eyepiece should be designed first, and the objective achromatism should then be arranged to include the axial chromatic aberration of the eyepiece. If the specification of the eyepiece is unknown, the range for the minimum wave-length should be taken as being about 505 to 535 so as to allow for the lowering effect of an unachromatic eyepiece.

Definition Tests in Daylight

It would appear at first sight that chromatic aberration can have little to do with sharpness of definition, yet in reality there is a considerable connection between them. The reason for this is that, to the eye, the most luminous part of the daylight spectrum appears at a wave-length about 555, and, if a telescope has its minimum focus at, say, 520, then the eye will be most sensitive to a wave-length which is considerably off the peak of the colour-curve of the instrument. Hence that part of the whole image which is most visible to the eye is not that which is focussed most nearly in one plane, and a uniform blurring is seen, quite independent of the presence or absence of coloured fringes and haze.

Another aberration which may cause diffusion of the image is the chromatic variation of the spherical correction. In the "chromatic telescope" used for the present series of experiments, the marginal spherical aberration was always corrected for the *minimum-focus* wave-length, whatever that might be; consequently, once again, if the minimum is at 520, the spherical aberration will not be corrected for the light to which the eye is most sensitive. We should expect, therefore, that a minimum in the neighbourhood of 555 would give us the best all-round definition of a critical object. This is, however, not quite the case, as for practically all black-and-white objects a distinct and somewhat objectionable purple haze is noticeable at this adjustment. At a very high minimum wave-length, say 590, in daylight, the image is misty and strongly blue; at about 555 it is sharp but still somewhat blue; at about 520 the colour disappears, but the image is not so clear as it was; and below 520 the definition falls off very considerably, and a greenish or reddish haze makes its appearance.

A series of tests similar to those for least colour was now performed with the aim of securing the sharpest possible definition in the image. Several observers were employed and the results were tabulated as before (see Table II).

From this series of observations it would appear that, to obtain an image having the clearest possible definition for objects in daylight, the minimum focus should be at a wave-length between 530 and 550. This applies to instruments of precision such as theodolites, levels, gun sights, and sighting telescopes generally. To allow for an uncorrected eyepiece, the objective should have its minimum somewhere between 545 and 565; the average figure 555, which arises when the *C* and *F* lines are combined at the focus of the ·7 zone, may therefore be regarded as very suitable for the achromatising of objectives for these purposes.

TABLE II. *Settings for Best Definition*

Type of object	High contrast						Low contrast					
Eyepiece power per inch	13			28			13			28		
	Obs.	Av.	No. of Obs.	Obs.	Av.	No. of Obs.	Obs.	Av.	No. of Obs.	Obs.	Av.	No. of Obs.
Sunlight	*A*	539	7	*A*	542	6	*A*	552	7	*A*	528	4
	B	547	4	*B*	532	5	*C*	535	1	*B*	537	3
	E	550	6	*C*	544	5	*M*	541	1	*C*	537	1
				E	550	3						
				F	537	8						
Highest individual setting		567			555			565			559	
Average setting		**545**	17		**540**	27		**549**	9		**532**	8
Lowest individual setting		529			513			535			515	
Shade	*A*	538	5	*A*	549	6	*A*	535	3	*A*	538	3
	B	532	5	*B*	543	5	*B*	540	3	*B*	542	3
	G	523	1	*C*	554	5	*E*	527	4	*C*	524	2
				H	527	3	*G*	523	1	*E*	545	7
Highest individual setting		550			569			562			558	
Average setting		**534**	11		**545**	19		**532**	11		**540**	15
Lowest individual setting		523			510			497			511	

GAS-FILLED LAMP TESTS

The "object" for these series of tests was a pictorial lantern slide of a black tone, erected in front of a 40-watt "Fullolite" lamp at a distance of 20 m. from the telescope. Various observers were employed as before, and they were asked to set the compensator to give either least colour over the whole picture or best

TABLE III. *Settings with Gas-filled Lamp*

Test	Least colour						Best definition					
Eyepiece power per inch	13			28			13			28		
	Obs.	Av.	No. of Obs.	Obs.	Av.	No. of Obs.	Obs.	Av.	No. of Obs.	Obs.	Av.	No. of Obs.
	A	509	10	*A*	522	23	*A*	562	11	*A*	559	14
	B	522	4	*B*	539	4	*B*	560	8	*B*	562	4
	F	504	5	*D*	520	14	*C*	552	2	*C*	556	10
	G	504	4	*E*	523	2	*F*	554	5	*E*	559	3
				F	513	13				*F*	562	3
				G	520	3				*G*	562	4
Highest individual setting		558			549			575			579	
Average setting		**509**	23		**520**	59		**559**	26		**559**	38
Lowest individual setting		465			502			531			552	

definition of some of the more critical details. The results are shown in Table III; they agree in general arrangement with those for daylight, but are mostly higher, especially those for best definition. This is to be expected, as the light of a gas-filled

lamp contains relatively more red and less violet than daylight, the peak of the luminosity curve being at about 566.

In addition to these observations on a black-and-white object, one or two coloured objects were used for definition tests. The results were as follows:

	Obs.	Av.	No. of Obs.	
Page of figures behind "daylight" glass	*A*	545	5	at 28 per inch
Microscope slide, stained rose-red	*A*	556	6	
Microscope slide, stained rose-red	*B*	573	5	
Figures, behind a microscopist's blue-green filter	*A*	526	2	at 13 per inch

It was very evident, in making these settings, that the minimum wave-length should bear some relation to the colour of the object; that is, a red slide requires a high minimum whereas a blue slide requires rather a low one, and so on. If a microscope were required for work with a certain specified stain, the best achromatism could easily be determined beforehand with this apparatus.

I suggest here that if anyone could ever spare the time to design a microscope objective to be used with a "chromatic plate" such as the compensator used in the present work, the results obtained with various microscopical objects and stains would be of immense value to the designers of microscopes generally. The figures here must necessarily be regarded as only approximate, in so far as they were obtained with a telescope instead of a microscope.

Tests with Astronomical Objects

For this work, by kind permission of Prof. Dingle, the telescope was attached to the $5\frac{1}{2}$-inch equatorial on the roof of the College; and settings were made on the Moon, Mars, and several stars. The figures obtained were much higher than for daylight work, this being probably due to the relative faintness of celestial objects compared with sunlight, whence the irritating purple halo, so obvious when daylight is used, becomes almost too faint to notice unless very bad indeed.

Before interpretation of the settings was possible, the instrument had to be calibrated for an infinitely distant object, and this was done by first obtaining the mercury and sodium calibration points at three finite distances (13, 20, and 65 metres respectively). As was expected, it was found that change of object-distance merely produced a linear shift of the calibration curve without altering its shape appreciably. By then plotting the amount of this shift against the reciprocal of the object-distance, a very small extrapolation gave the position of the calibration curves for an infinitely distant object. This shift was found to be ·5 cm. towards the objective.

The two original eyepieces were used for this work also, together with a 4 mm. Beck microscope objective, mounted in a tube to act as a Ramsden eyepiece giving a power of 50 per inch aperture. The calibration curve for this is also shown in Fig. 6.

The Moon. Settings on the Moon gave a best minimum at about 565, the tolerable range being from 540 to 590. These figures were identical at 35 and 50 per inch powers. At lower powers the range became larger. Too low a minimum gave a red fringe with general loss of definition; too high a minimum gave a distinct violet halo.

Mars. The average settings at 13, 28, and 50 per inch powers were all about 574 (extreme range 550 to 590). As is always the case, higher minima give a purple or violet halo, and lower values give fuzzy red or green images.

Rigel. Average setting 570 at 28 and 50 per inch (range 560–580).

Aldebaran. Average setting 573 at 50 per inch (range 554–581).

Castor. Average setting 579 at 50 per inch.

With faint stars it appears to be quite immaterial what type of chromatic correction is used.

As far as the present experiments have shown, the ideal minimum wave-length for small astronomical telescopes is about 570 to 575, although anywhere inside the range 565 to 580 would be entirely acceptable. For designers who use the older method of tracing rays in two colours, it may be mentioned that uniting the *B* and *F* rays at the focus produces a minimum at about 565.

I am greatly indebted to Mr W. D. Wright for his kindness in assisting with these observations.

It is only fair to add that the two observers who made these settings are novices at astronomical work, and that an experienced observer would probably make somewhat different settings. On the other hand, we have the advantage of being in no way biassed by familiarity with any one particular state of achromatism, as an astronomer would undoubtedly be. Yet the long experience of an astronomer in looking for real critical definition on difficult objects would give him a very definite opinion as to the nature of the most desirable image, which would be of immense value to one who is making settings with this chromatic telescope. If it ever becomes possible to secure the services of a skilled astronomer on a fine evening to make some settings with this apparatus, I shall do my best to arrange it.

Chromatic Difference of Magnification

There is one other use to which the chromatic telescope may be put, and that is to determine the best state of oblique achromatism (or achromatism of magnification) for an eyepiece. As is well known, the two lenses of a Huygenian eyepiece are so chosen that a white ray entering the field-lens emerges from the eye-lens with two of its constituent colours (usually red and blue) parallel. As is also the case with axial achromatism, the fact that two colours have been made to conform to a desired condition does not ensure that other colours will also obey that condition. Actually in an eyepiece there will be a lateral secondary spectrum effect remaining when two colours have been brought to strict parallelism. Somewhere between these two colours there will be a wave-length for which the magnification is a maximum, and it was thought that the present apparatus could be adapted

to enable determinations of the best value for this "wave-length of maximum magnification" to be made.

The "object" for this test was a slit, 25 mm. by 3 mm. in size, made in a piece of card held in front of a "Fullolite" lamp. In order to reproduce the conditions existing in the outer field of an eyepiece, only a narrow eccentric portion of the objective was used to form the image. This portion was isolated by a card screen attached in front of the objective, having a rectangular aperture, 20 mm. by 6 mm. in size, situated in the ·7 zone to which the calibration curves of the apparatus applied. This screen was so placed that the long side of the aperture was parallel to the illuminated slit. The image seen under these limitations was a narrow bar of light in the centre of the field with coloured bands along each side of it, the colours being of course different on the two sides. By moving the compensator it was possible to find a position at which the image itself was practically colourless and the coloured fringes were almost invisible. The minimum wave-length corresponding to this position was read off and was found to be 545.

Owing to the uncorrected chromatic aberration of the eye, the greatest care was necessary in making these settings to ensure that the image was seen in the exact centre of the visible field. This was secured by placing the eye about two inches away from the eyepiece, and letting the bar of light appear exactly in the centre of the exit-pupil. When designing an eyepiece, therefore, it should be the designer's aim to secure maximum magnification for approximately this wave-length—545 for electric light, or about 530 for daylight.

My best thanks are due to Prof. A. E. Conrady, who suggested this research and gave great assistance, both in the design of the apparatus and throughout the experimental work; to the proprietors of Messrs Wray, Ltd., who took great trouble to make the objective and compensator with sufficient accuracy; and, by no means least, to those many people who kindly offered their time and services as observers to make the settings, frequently at considerable personal inconvenience.

DISCUSSION

Comdr N. M. S. Langlands: Mr Kingslake's paper is obviously of very great importance and is a further valuable contribution from the Applied Optics Department of the Imperial College.

In the laboratory one uses test-objects which are built up of well-defined points and lines. Has Mr Kingslake tested his instrument on objects with natural backgrounds such as one might find when sighting on a target? In such cases one might have to take into account the use of filters. The size of the exit-pupil does not appear to be given in the paper. The individual variation in setting depends not only on the simple chromatic aberration of the observer's eye but on the lack of perfect centering of the refracting surfaces of the eye, and the diameter of the exit-pupil of the telescope and its position relative to the entrance-pupil of the observing eye must be of some importance.

Prof. L. C. Martin: I should like to express my appreciation of Mr Kingslake's valuable paper. I believe it is the case that Fraunhofer did not adopt the method of achromatism used by him without a careful investigation, and it would have been of interest if Mr Kingslake had added some reference to this early work which came so near the results of later experience.

While the difficulty of obtaining a large number of observations by different observers is very great, would not the experiments have been more valuable if they had not been so purely "subjective"? If an observer had something unknown to recognise in each observation and he succeeded say in 40 per cent. of his observations when the minimum focus was at G', but in 80 per cent. when at E, one would have obtained a type of statistical result which would have been more satisfying, to me at least, than results which depend on qualitative estimations.

The type of apparatus which Mr Kingslake has developed would lend itself admirably for purposes of investigating the kind of achromatism for use with various stained objects in microscopy, a subject on which very little detailed information is available.

Mr W. D. Wright: There is one point which Mr Kingslake has not made clear in his paper but must be properly grasped before a correct interpretation of the results can be obtained, namely, that the minimum wave-length which is being determined is the wave-length of the rays coming to the shortest focus *in the eye*. Hence, when it is required to find the most desirable minimum wave-length of, say, a telescope objective, the chromatic aberration both of the eyepiece *and* of the eye must be taken into account in working back from Mr Kingslake's results. This also introduces the possibility of discrepancies due to different observers having somewhat different chromatic aberration.

In this connection it is interesting to note the consistent difference between the results for observers A and B in all the settings for least colour. In every case B requires a longer minimum wave-length than A. Inasmuch as this difference does not occur in the results for best definition, at least not with the same consistency, the cause is most probably a difference of sensitivity for the blue end of the spectrum. The possibility of the chromatic aberration of the eye being different for different observers should not be overlooked, however, and it may be pointed out that a separate calibration curve for each observer would eliminate any disagreements from this cause.

Mr T. Smith: The scale of the instrument, as distinct from its type, will influence the type of correction desired. Assuming that the eye in combination with an eyepiece would make the best of the light brought to a focus between two definite planes and as far as possible disregard the rest, it is apparent that with instruments of long focal length the spherical aberration must be corrected for just about the frequency for which the focal length was stationary, but that conclusion does not appear to hold necessarily for small instruments. The fact that the best correction for freedom from colour found in these experiments disagrees with that for definition might be taken as *prima facie* evidence that these two

corrections should preferably be made for different frequencies. Further experiments in which the frequency for spherical correction could be made to depart arbitrarily from that for which the minimum focussing distance was secured should remove any uncertainty on the best type of central correction.

It would be interesting to know how far the characteristic chromatic aberration curves for the system employed in this investigation, in which the separation between the objective and compensator is large, depart from those for normal objectives where the overall axial length is small. Any difference there might be appears to be small, but it would be as well to know from calculation that it is practically negligible.

Prof. A. F. C. Pollard: Mr Kingslake is to be congratulated not only on the interesting and important paper he has read to us this evening, but also on the able and clear manner in which he has presented it.

When the indications of an instrument depend upon the "judgment" of the measured quantity by the observer, we should be particularly careful to see that this judgment is not interfered with or biassed by other judgments conscious or unconscious. The elimination of such "disturbances" in the "field of observation" is characteristic of a good experiment. We cannot always get rid of these disturbances and sometimes we can even make use of them.

When Mr Kingslake asked me to make some observations for him, I pointed out that, if he wished to get more consistent and reliable observations, it would be necessary to re-design the mechanism of his telescope in order that the settings could be made without any possible knowledge of the previous setting entering as a "judgment" over successive settings in the course of a series of observations. He should be particularly careful that his "judgment" of the appearance of the image is not disturbed by his possibly unconscious judgment of the setting of the indicator of the instrument. Personally I found this condition difficult to realise. Even if a second person operates the setting, so acutely apprehensive are we of movement, noise, and such impressions which may fall upon our sense organs, he should be screened from the observer.

In short, we should regard ourselves as complex instruments, difficult to understand in more ways than one, and when we desire to use a particular judgment for the purpose of observation it is almost imperative that we make a close study of our natural responses to stimuli and more particularly to those responses which constitute "disturbances" in the "field of observation" during an experiment—and this is a much neglected and troublesome matter.

Mr Kingslake: In reply to Comdr Langlands, I was practically forced to use artificial rather than natural objects for the observations described in this paper, on account of the necessity of maintaining a constant object-distance, and for convenience in the experimental arrangements. On a few occasions I have made trial settings on small objects against the sky and also on distant buildings, etc., and it would probably be of interest if further settings of this kind could be made. Comdr Langlands suggests a few causes for the difference between settings by

different observers, but he has omitted one which is probably as important as any, namely, the sensitiveness of the eye to the blue end of the visible spectrum. In my own case, for example, I object very strongly to a trace of blue haze in a prismatic binocular, where another user would not notice any colour whatever. The size of the exit-pupil is implied in the stated "power per inch" of the various eyepieces used, since a power of 13 per inch necessarily produces an exit-pupil $\frac{1}{13}$ inch in diameter.

With regard to Prof. Martin's reference to Fraunhofer, it appears* that he achromatised his telescope objectives by assigning a value to the ratio of the net curvatures of the two components. He originally determined this ratio by dividing the spectrum into regions bounded by the successive Fraunhofer lines *A*, *B*, *C*, etc., and then measuring the ratio of the flint dispersion to the crown dispersion for each region. Owing to the unequal dispersions of the two types of glass, these ratios for the different regions of the spectrum were somewhat different, so he then adopted a mean of all the ratios, weighted according to the amount of light included in each region. Owing to the crowding together of the red end of the prismatic spectrum, this process would ultimately give an achromatic lens with its minimum focus colour somewhere towards the blue, and he then found empirically that if he altered the ratio of total curvatures slightly, he obtained an image which gave better definition although somewhat coloured. In the terminology of the present paper, he preferred a minimum focus more towards the red than that which he had originally determined by means of his method of the weighted mean dispersion-ratio. The idea of securing achromatism by uniting two wave-lengths at the focus appears to have been a later development which arose when accurate ray-tracing at finite aperture became more general.

In many ways subjective tests are rather unsatisfactory to a scientific observer, but in other ways they are of great value. In the present case the aim of the work was mainly to find out what sort of image most satisfied the various observers, in order to produce a telescope which should most satisfy the user. It seemed therefore that subjective experiments were necessary to fulfil this aim. In any case tests on the lines suggested by Prof. Martin would only be of value in determining the best achromatism for good resolution, and not for least colour in the image.

Mr Wright has emphasised the effect of an unachromatic eye. If an observer were to determine a calibration curve and then perform a number of settings, the results would be true for the ultimate image on his retina, and, since different eyes might quite easily have different amounts of chromatic aberration, it is evidently advisable for each observer to perform his own calibration. But, as the errors introduced in this way would be very small compared with the extent of the range of the settings, it was not thought necessary to press this point. It may be of interest to mention that, although observers *A* and *B* differ consistently by about 20 units of wave-length in the least-colour settings, their calibration curves only differ by about 5 units.

If an apparatus could be constructed in which the chromatic aberration and

* Czapski-Eppenstein, *Theorie der optischen Instrumente* (305).

the wave-length of spherical correction could both be altered independently, then Mr T. Smith's interesting point could be explored. It is quite likely that some very useful discoveries might be made on these lines.

I agree with Prof. Pollard that the setting should be operated by a handle which precludes any possibility of remembering the last settings. In some cases another person made the settings, but this was found to be just as unsatisfactory because then the setter would remember past settings and the movements he would give to the sliding lens would be similarly biassed.

THE USE OF PHOTO-ELECTRIC CELLS FOR THE PHOTOMETRY OF ELECTRIC LAMPS

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ABSTRACT. A description is given of apparatus and experiments designed for giving the highest accuracy and precision in the photometry of electric lamps, using photo-electric cells. Although the methods adopted are not novel, yet every care has been taken to obtain the highest sensitivity in the photo-electric current measuring apparatus and to maintain the lamps at a steady accurately known voltage. It is claimed, therefore, that the results are useful in showing the maximum capabilities of photo-electric cells when used in the usual manner with a sensitive electrometer or electroscope.

The actual results may be briefly stated as follows:

	Photo-electric per cent.	Visual per cent.
Accuracy of mean spherical colour matching	0·1 voltage	0·5 voltage
Accuracy of mean spherical intensity measurements	0·2 intensity	0·4 intensity
Accuracy of directional colour matching	0·1 voltage	0·5 voltage
Accuracy of directional intensity measurements	0·15 intensity	0·25 intensity

A discussion is given of the sources of error to be expected, of the method by which they may be avoided, and of the computation of the accuracy of the results.

Introduction

In a previous paper* a description of apparatus designed for the colour matching of electric lamps and for intensity measurements of them, using photo-electric cells for these purposes, has been given. Some of the earlier measurements made with this apparatus were also recorded. Since then a large number of measurements have been made with this apparatus. The lamps used for this purpose are of the usual standard pattern with the filaments all in the same plane. This facilitates alignment of the lamps either on the photometer bench or in the cube. The Dolezalek electrometer has been replaced by a Compton electrometer with a rolled out Wollaston wire suspension which gives a sensitivity of 1800 mm. per metre per volt with a periodic swing of 7 seconds. Much work has been done by several investigators† with regard to the uses of photo-electric cells for photometry, but

* T. H. Harrison, *Proc. Opt. Convention*, 1926 (245).

† J. Elster and H. Geitel, *Phys. Z.* **10** 1909 (477), **11** 1910 (251, 1212), **12** 1911 (609), **13** 1912 (739); H. E. Ives, *Astrophys. Journ.* **39** 1914 (428), **43** 1916 (9); J. Kunz, *ibid.* **45** 1917 (69); A. F. and F. A. Lindemann, *Monthly Notices, R.A.S.* **79** 1918 (343); K. S. Gibson, *Journ. Opt. Soc. Amer.* **2** 1919 (23); G. Rougier, *Rev. d'Opt.* **2** 1923 (365); E. A. Baker, *Journ. Scient. Instr.* **1** 1923–4 (345); N. R. Campbell, *ibid.* **2** 1924–5 (177), **3** 1925–6 (2); C. H. Sharp and C. Kinsley, *Trans. Ill. Eng. Soc. New York*, **21** 1926 (117).

in most cases they have aimed chiefly at securing speed of working and the elimination of the eye, involving as it does the personal factor of the observer; moreover, most of them have been content with the ordinary accuracy sufficient for the rating of lamps in a lamp works.

N. R. Campbell* and his collaborators at the Research Laboratories of the General Electric Co., Ltd., Wembley, have shown, however, that photo-electric methods are capable of giving results which can be repeated more accurately than those obtained by visual methods, and have moreover used them successfully in studying the behaviour of tungsten filaments at vacuum lamp temperatures. The results of Campbell's earlier work on this subject suggested that an investigation with regard to the highest precision in photometry obtainable by the photo-electric method in a standardising laboratory would be of value.

The work described in the present paper is in the nature of such an investigation in which precision is the chief aim, no attention being paid to speed of working.

The results obtained with the apparatus employed, in which every care has been taken to ensure the highest possible sensitivity and precision, are discussed in this paper, and although the methods adopted are not novel, yet it is thought that the work described may be of value as indicating what, in the author's opinion, is the highest sensitivity and accuracy that can be obtained with photo-electric cells when used in the manner described here.

The work can conveniently be divided into four sections:

(*a*) Colour matching of lamps for mean spherical colour.
(*b*) Mean spherical intensity measurements.
(*c*) Colour matching of lamps for directional colour.
(*d*) Directional intensity measurements.

General Principle of Apparatus

An electric lamp is placed in a 4 ft. cube at the bottom of which is a photo-electric cell exposed to the light of the lamp reflected from the walls of the cube. At the end of a photometer bench, on which another lamp is placed, there is a second photo-electric cell exposed to the directional light from this lamp. The two photo-electric cells are connected in series and form two arms of a Wheatstone bridge. A Compton electrometer measures the difference between the two photo-electric currents. By placing test lamps in turn in the cube and obtaining a balance by moving a comparison lamp along the bench, one can obtain relative mean spherical intensities of the lamps in the cube, whilst by placing test lamps in turn on the bench and finding the balance positions as before, one can obtain the directional intensities of these lamps.

The test lamps must in general be colour matched with each other since the photo-electric cells do not have the same light-frequency sensitivity curve as that of the normal eye. In order to render this colour matching less necessary, it is desirable to use a rubidium cell rather than a potassium or sodium one, and, further,

* N. R. Campbell, *loc. cit.*

to use in front of it a suitable yellow filter to cut out the excessive sensitivity of the cell in the blue region. A Wratten K 3 filter has been used, and it has been found that an error in colour match corresponding to a 1 per cent. change in voltage on the lamp when the latter is run at an efficiency of about 1·5 watts per candle (directional) leads to an error in candle-power measurement of only 0·1 per cent.

For colour matching, a rubidium cell (with or without a yellow filter) and a sodium cell are placed as close together as possible either at the bottom of the cube or at the end of the bench, and by virtue of the fact that the two cells have different characteristic curves, colour matching can be carried out in the manner described by Campbell*.

In order to align the lamps correctly in the cube, two small holes are bored, one in the middle of one of the sides and the other in the middle of the back wall. These holes are covered with suitable converging lenses to give images by means of which the lamp position can be set to within a millimetre.

The Cells

The photo-electric cells used in the work described were generally of the vacuum cathode plate type made at the G.E.C. Research Laboratories at Wembley, but neon-filled cells of the anode ring type made at the National Physical Laboratory have also been successfully used. In general the latter type are rather more sensitive but perhaps not quite so steady in their behaviour.

Since all photo-electric cells when illuminated are subject to a gradual change in sensitivity dependent on the flux of light falling into them it has been found necessary to measure the lamps in a certain cyclic order and also, as far as the mean spherical properties of lamps are concerned, to deal with lamps of approximately the same intensity. The reason for the latter restriction is that unless the flux of light falling into the cell is approximately constant the change in sensitivity of the cell (photo-electric fatigue) is not linear and consequently no ordinary cyclic procedure will eliminate the effect. Another reason why lamps of approximately the same intensity should be used is that the current in photo-electric cells employing alkali metals is not strictly proportional to the flux of light producing it. This lack of proportionality, however, proved, with the particular cells used, to be less serious than the "fatigue" effect.

The Lamps

Two sets of lamps have been used, one set consisting of four nominal 100 volt 30 watt lamps and the other of four nominal 100 volt 75 watt lamps. Lamps of the first set will be lettered *A*, *B*, *C*, *D* and those of the second set *E*, *F*, *G*, and *H*. The cyclic order, followed in a series of readings in order to eliminate, as far as possible, the effect of photo-electric fatigue, was *A D C B A B C D A D C B A*, and the first four out of these thirteen readings were neglected, this being done to ensure that the apparatus had settled down to a steady behaviour. This leaves

* N. R. Campbell, *loc. cit.*

three readings for lamp *A* and two for each of the lamps *B*, *C*, *D*. The mean of the readings for each lamp is then taken and considered to be the result of the set of readings. The same procedure is followed for lamps *E*, *F*, *G*, and *H*.

Colour Matching of Lamps for Mean Spherical Colour

A rubidium cell and a sodium cell were placed at the bottom of the cube and the rubidium cell covered with an adjustable filter wedge which was adjusted so that a balance of photo-electric currents occurred when a lamp in the cube was run at a suitable efficiency, in this case 1·53 watts per candle (directional). Lamps *A*, *B*, *C*, and *D* were then inserted in the cube in the order indicated in Table I which shows a typical example of a set of readings. The lamps were correctly positioned and the voltage on them was then adjusted so that the photo-electric currents were equal, as indicated by zero deflection of the electrometer needle.

Table I. *Mean Spherical Colour Matching by the Photo-electric Method*

Lamp	Colour match voltage	Relative colour match voltage adjusted to mean value of 100
A	96·52	—
D	97·94	—
B	96·69	—
C	98·99	—
A	96·51	98·95
B	98·99	101·45
C	96·71	99·12
D	98·05	100·49
A	96·76	—
D	98·25	—
C	96·93	—
B	99·17	—
A	96·77	—

In order to find out the possible errors due to failure to align lamps accurately inside the cube, the lamp was displaced in different directions and the error noted. The results are given in Table II.

Table II

Displacement	Error per cent. voltage
1 cm. vertical	0·40
1 cm. back to front	0·30
1 cm. right to left	0·20
Rotation of about 30°	0·15
Tilt of about 5°	0·20

The total error, if the lamp were displaced by all the above amounts, would be 1·25 per cent. voltage. When attempting to align the lamps correctly, however, it is most unlikely that the displacements would be as much as $\frac{1}{10}$ of the above

amounts, and the error due to incorrect positioning of the lamps would not exceed 0·1 per cent. voltage.

In all, four sets of readings such as that shown in Table I have been made with lamps *A*, *B*, *C*, and *D* and another four with lamps *E*, *F*, *G*, and *H*. These are shown in Table III.

Table III. *Mean Spherical Colour Matching by the Photo-electric Method*

Lamp	Relative voltage for colour match	Maximum difference	Lamp	Relative voltage for colour match	Maximum difference
A	98·95	0·10	*E*	100·55	0·08
	98·93			100·63	
	98·95			100·62	
	98·85			100·63	
B	101·45	0·09	*F*	100·91	0·08
	101·47			100·94	
	101·54			100·93	
	101·52			100·86	
C	99·12	0·12	*G*	98·30	0·04
	99·13			98·33	
	99·07			98·30	
	99·01			98·29	
D	100·49	0·19	*H*	100·25	0·15
	100·46			100·10	
	100·42			100·14	
	100·61			100·23	

It will be seen from Table III that the consistency of the readings is of a high order, and it appears very unlikely that there can be an error in the mean value of more than 0·1 per cent. voltage.

In order to find how the results of the photo-electric method agreed with those of the visual method, two sets of visual mean spherical colour matching measurements were made upon the same lamps and the results are shown in Table IV.

Table IV. *Mean Spherical Colour Matching by the Visual Method*

Lamp	Relative voltage for colour match	Difference	Lamp	Relative voltage for colour match	Difference
A	98·9	0	*E*	100·7	0·1
	98·9			100·8	
B	101·0	0·1	*F*	100·2	0·5
	101·1			100·5	
C	99·1	0·5	*G*	98·9	0·2
	98·6			98·7	
D	101·1	0·2	*H*	100·3	0·5
	101·3			99·8	

The error in the most accurate visual colour matching is usually put at 0·5 per cent. voltage and the above figures bear this out fairly well.

A comparison between the relative spherical colour match voltages of the lamps as measured photo-electrically and visually is shown in Table V.

From Table V it may be observed that the differences between the photo-electric and visual results are rather higher than one would expect them to be if they were due only to experimental errors. The cause of these errors is discussed later in the paper.

Table V. *Comparison of the Relative Mean Spherical Colour Match* Voltage as measured (a) photo-electrically, (b) visually*

Lamp	Relative voltage for colour match		Difference
	(*a*)	(*b*)	
A	98·92	98·8	+ 0·12
B	101·50	101·0	+ 0·50
C	99·08	98·8	+ 0·28
D	100·50	101·2	− 0·70
E	100·61	100·7	− 0·09
F	100·91	100·4	+ 0·51
G	98·31	98·8	− 0·49
H	100·18	100·0	+ 0·18

In view of the divergence of opinion as to whether lamps should be rated by volts or by amperes and in view of the fact that, if variable resistances at the contacts occur inside the lamp rating by amperes would be the better method, both methods have been used in these experiments. It has been found that, for the lamps used, the consistency of the measurements was the same for both methods. There appeared therefore to be no reason for reversing the procedure, usually adopted in this laboratory, of observing the voltages on lamps continuously throughout photometric measurements and measuring the current occasionally.

Mean Spherical Intensity Measurements

For these measurements one rubidium cell covered with a Wratten K 3 filter was placed at the bottom of the cube and another cell placed at the end of the photometer bench. The same cyclic process in a set of readings as described above was gone through and the bench reading for a balance of two photo-electric currents gave a measure of the mean spherical intensity of the test lamps in the cube. Table VI shows the results of these sets of readings. The voltages used on the lamps were those found to give a colour match by the photo-electric method.

The mean spherical intensity measurements by the visual method, made under conditions similar to those for the measurements in Table VI, are given in Table VII.

A comparison of the results of the photo-electric and visual methods is given in Table VIII.

It is rather difficult to estimate the accuracy of the results. The first two sets of photo-electric measurements are very consistent.

* The inaccuracy of the photo-electric results is estimated at 0·1 per cent., while that of the visual is about 0·5 per cent.

Table VI. *Mean Spherical Intensity Measurements by the Photo-electric Method*

Lamp	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Volts on lamp	97·00	99·50	97·17	98·53	94·00	94·29	91·84	93·61
Amperes through lamp	0·28104	0·28059	0·28135	0·27674	0·70726	0·71194	0·70357	0·70707
Relative intensities 1st set	101·04	102·07	100·84	96·00	101·04	101·98	95·98	100·95
2nd set	101·10	102·08	100·71	96·05	101·00	102·05	95·91	101·00
3rd set	—	—	—	—	100·94	102·50	95·76	100·76
4th set	—	—	—	—	100·93	102·38	96·00	100·65
Mean	101·07	102·08	100·78	96·03	100·98	102·23	95·91	100·89
Maximum difference	0·06	0·01	0·13	0·05	0·11	0·52	0·24	0·35
Relative volts	98·92	101·50	99·08	100·50	100·61	100·91	98·30	100·18
Relative amperes	100·40	100·22	100·50	98·87	99·98	100·64	99·44	99·94
Relative efficiency (watts per mean spherical candle-power)	98·19	99·57	98·74	103·55	99·54	99·49	101·86	99·11

Table VII. *Mean Spherical Intensity Measurements by the Visual Method*

Lamp	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Volts on lamp	97·00	99·50	97·17	98·53	94·00	94·29	91·84	93·61
Amperes through lamp	0·28104	0·28059	0·28135	0·27674	0·70726	0·71194	0·70357	0·70707
Relative intensities 1st set*	101·08	102·03	100·83	96·04	100·92	102·05	95·91	101·08
2nd set*	101·12	102·13	100·52	96·20	100·53	102·25	96·28	100·91
3rd set*	101·51	102·08	100·21	96·15	100·40	102·49	95·88	101·22
Mean	101·24	102·08	100·52	96·13	100·62	102·27	96·02	101·07
Maximum difference	0·43	0·10	0·62	0·16	0·52	0·44	0·40	0·31
Relative volts	98·92	101·50	98·06	100·50	100·61	100·91	98·30	100·18
Relative amperes	100·40	100·22	100·50	98·87	99·98	100·64	99·44	99·94
Relative efficiency (watts per mean spherical candle-power)	98·04	99·60	99·02	103·34	99·94	99·27	101·76	99·02

* Mean of results obtained by two observers.

The visual measurements were then made, after which the third and the fourth sets of photo-electric readings were made in order to verify the results. They do not agree with the previous sets of photo-electric readings as well as was expected, but it is thought that there may have been a change in the candle-powers of the lamps, since about a month had elapsed between the measurements. It is certainly strange that while results of measurements made on the same day are in very close agreement (usually within 0·1 per cent.) those made at some other time give results which may differ by 0·3 per cent., yet this has been observed by the visual method as well as the photo-electric method. Lamps of the 75-watt

Table VIII. *Comparison of the Mean Spherical Intensity as measured (a) photo-electrically, (b) visually*

Lamp	Relative intensity		Difference
	(*a*)	(*b*)	
A	101·07	101·24	− 0·17
B	102·08	102·08	0
C	100·78	100·52	+ 0·26
D	96·03	96·13	− 0·10
E	100·98	100·62	+ 0·36
F	102·23	102·27	− 0·04
G	95·91	96·02	− 0·11
H	100·89	101·07	− 0·18

type such as *E*, *F*, *G*, and *H* are known from visual tests to be not so reliable as those of the 30-watt type such as *A*, *B*, *C*, and *D*. In finding that the candle-powers of these lamps vary slightly, the photo-electric measurements confirm and measure more precisely the differences which are known to exist from cumulative visual results. It is most unlikely that the photo-electric results are in error by more than 0·2 per cent., whilst the accuracy of the visual is about 0·5 per cent. The differences shown in Table VIII between the visual and photo-electric results are obviously due only to experimental error, that is, if the precision of both types of measurements were to be increased, these differences would on the average decrease proportionally.

Directional Colour Matching

For the directional photo-electric colour matching, a rubidium cell with yellow filter and a sodium cell were placed as close together as conveniently possible at the end of the photometer bench. The lamps were placed about 160 cm. from the cells. In order to compensate for the different distribution curves of the lamps it was necessary to interchange the positions of the two cells and take the mean of the results. Two sets of readings were made with the cells in one order and two with the positions of the cells reversed. In order to even out the light a plate of slightly diffusing glass was placed just in front of the lamps. The results of these sets of readings, together with those of two sets of visual measurements, are shown in Table IX.

Table IX. *Directional Colour Matching (Photo-electric and Visual)*

Lamp	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Photo-electric readings								
Cells in 1st position 1st set	98·43	101·57	98·75	101·26	100·76	101·40	98·29	99·56
2nd set	98·38	101·47	98·79	101·34	100·77	101·30	98·33	99·51
Relative colour match volts								
Cell positions reversed 1st set	98·69	101·30	98·81	101·18	100·27	101·16	98·65	99·92
2nd set	98·65	101·26	98·94	101·14	100·32	101·08	98·74	99·85
Maximum difference of readings of same character	0·05	0·10	0·13	0·08	·05	·10	·09	·07
Mean ± 0·1	98·54	101·40	98·83	101·23	100·52	101·24	98·49	99·74
Visual readings 1st set	98·1	101·2	98·8	102·0	100·8	100·9	98·7	99·4
2nd set	97·9	101·7	98·6	101·7	100·7	101·6	98·7	98·9
Difference	0·2	0·5	0·2	0·3	0·1	0·7	0·0	0·5
Mean ± 0·5	98·0	101·5	98·7	101·9	100·8	101·3	98·7	99·2
Difference between photo-electric and visual results	+ 0·5	− 0·1	+ 0·1	− 0·7	+ 0·3	− 0·1	− 0·2	+ 0·5

Table X. *Directional Intensity Measurements*

Lamp	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Volts on lamp	97·00	99·81	97·29	99·67	94·00	94·67	92·11	93·27
Amperes through lamp	0·28125	0·28148	0·28193	0·27881	0·70812	0·71427	0·70481	0·70646
Photo-electric relative intensities								
1st set	99·93	100·28	100·11	99·67	100·80	102·79	97·53	98·89
2nd set	100·12	100·30	100·04	99·52	100·69	102·69	97·50	99·12
Difference	0·19	0·02	0·07	0·15	0·11	0·10	0·03	0·23
Mean	100·03	100·29	100·08	99·60	100·75	102·74	97·52	99·01
Candle-power	17·856	17·899	17·872	17·781	43·550	44·400	42·160	42·805
Watts per candle	1·5280	1·5699	1·5350	1·5632	1·5285	1·5133	1·5399	1·5396
Relative volts	98·54	101·40	98·83	101·23	100·52	101·24	98·49	99·74
Relative amperes	100·14	100·21	100·37	99·28	99·96	100·83	99·49	99·72
Relative watts per candle	98·65	101·35	99·09	100·91	99·90	98·90	100·62	100·60
Visual relative intensities	99·75	100·44	99·93	99·87	100·71	103·08	97·48	98·73
Candle-power	17·81	17·93	17·84	17·83	43·53	44·56	42·14	42·68
Watts per candle	1·5318	1·5669	1·5375	1·5585	1·5294	1·5175	1·5406	1·5439
Relative watts per candle	98·91	101·18	99·28	100·64	99·76	99·00	100·56	100·66
Difference between photo-electric and visual results	− 0·28	+ 0·15	− 0·15	+ 0·27	− 0·04	+ 0·34	− 0·04	− 0·28

It appears from Table IX, since the estimated error of the photo-electric results is not more than 0·1 per cent. voltage and that for the visual measurements is not more than 0·5 per cent., that the difference between the photo-electric and visual measurements may be slightly more than can be expected from experimental error. This can only be due to the breakdown of the fundamental assumption that all well constructed lamps with clear, colourless bulbs can be matched for energy-frequency distribution. This will be discussed later.

Directional Intensity Measurements

For these measurements a rubidium cell with a Wratten K 3 filter is placed at the end of the bench, and another cell is placed at the bottom of the cube. Two sets of photo-electric readings are taken for each set of lamps. The results of these measurements together with visual measurements are shown in Table X.

In the photo-electric section of Table X the actual candle-power is calculated from the relative intensities found photo-electrically, using the mean candle-power found visually.

As in the mean spherical intensity measurements, it would appear that the mean of the two sets of photo-electric readings gives an accuracy of about 0·2 per cent.

For the visual measurements two observers took readings on four different days. The mean error for each day was 0·4 per cent. and the final error of such a set of observations is usually taken to be within 0·25 per cent. On the whole therefore the differences between the photo-electric and visual measurements seem to be explainable as ordinary observational experimental errors, and in the writer's opinion the photo-electric measurements give the more reliable results.

Discussion on Colour Matching

The most surprising facts of the colour matching experiments are that the mean spherical colour matches are not the same as the directional and that the visual results do not agree with the photo-electric results quite as well as one would expect.

In Table XI the final results of all the colour matching experiments are collected for the sake of comparison, and the estimated accuracy noted.

Both the photo-electric and visual measurements show a difference between the mean spherical and the directional colour, which can be accounted for on the supposition that the colour of the glass of the lamps is not uniform. This gives a clue as to the reason for the difference between the photo-electric and visual results. Broadly speaking, this difference is and must be due to a failure in the fundamental assumption that electric lamps can be accurately matched for colour or, to be precise, for energy distribution. This failure may be due to three causes:

(1) Different emissivity functions for different samples of tungsten.
(2) Different cooling effects at the filament ends of different lamps.
(3) Different selective colour of the glass for different lamps.

The first of these is a subject very difficult to deal with and somewhat out of the scope of the present work. It could be investigated by using double filament lamps in which any effects of selective colour of the bulbs would be eliminated, although there might be effects due to differential blackening of the bulbs. This problem may be attacked in the near future. The second case has been investigated with a lamp which had two separate hair-pin filaments, one nearly four times the length of the other, the two filaments being of the same diameter. The difference between the photo-electric and visual colour match voltages for these two filaments was $1{\cdot}2 \pm {\cdot}75$ per cent. Considering the fact that the shorter of these filaments was only about 5 cm. long and was fairly thick, carrying a current of 0·87 amp.,

Table XI. *Comparison of the Relative Colour Match Voltage* for Mean Spherical and Directional Colour when used (a) photo-electrically, (b) visually*

Lamp	Relative voltage for mean spherical colour match		Relative voltage for directional colour match	
	(*a*)	(*b*)	(*a*)	(*b*)
A	98·92	98·8	98·54	98·0
B	101·50	101·0	101·40	101·5
C	99·08	98·8	98·83	98·7
D	100·50	101·2	101·23	101·9
E	100·61	100·7	100·52	100·8
F	100·91	100·4	101·24	101·3
G	98·31	98·8	98·49	98·7
H	100·18	100·0	99·74	99·2

this difference is very small and shows that, when dealing with lamps whose filaments are thin and long and approximately of the same length, any colour differences resulting from the end cooling effects are negligible. Thus the indication is that the colour of the glass bulbs is the source of the errors; for not only is this more to be expected than an effect due to the different emissivities of different samples of tungsten wire but the difference between spherical and directional colour shows that the glass has a noticeable effect.

It may be worth while to consider the effects of two other possibilities, namely:

(1) Heterogeneity of the paint of the cube.

(2) Differential blackening (as distinct from selective coloration) of the bulbs.

If differential blackening of the lamps is sufficiently marked, even if the lamps can be accurately matched for energy-frequency distribution, then, if the paint of the cube is sufficiently patchy in the sense that different parts are of different colours, one selective colour will, for a given lamp, tend to predominate. If another lamp, supposed to be of the same colour as the first, is then placed in the cube, its distribution, on account of differential blackening, will in general be different from the first, with the result that another selective colour will tend to predominate. It follows, therefore, that a combination of the two above-mentioned factors can produce a difference between photo-electric and visual spherical colour matching

* The inaccuracy of the photo-electric results is estimated at 0·1 per cent., while that of the visual is about 0·5 per cent.

results and a difference between spherical and directional results. Neither factor by itself can produce any effect. As far as the directional results are concerned the existence of either or both of these factors makes no difference to the results. But the difference between the directional photo-electric and visual colour match voltage shown in Table XI is about the same as that for the spherical, so it appears that the effects of heterogeneity of the paint and differential blackening of the lamps are not so marked as those of the selective coloration of the bulbs.

There are four tests which may be employed in the estimation of the accuracy of photo-electric colour matching.

(*a*) The consistency of the photo-electric results with one another.

(*b*) The differences between the photo-electric and visual results.

(*c*) The differences between the intensities when measured:

(i) with a cell filtered to reproduce approximately the sensitivity curve of the eye;

(ii) with a cell not filtered whose sensitivity curve is considerably different from that of the eye.

(*d*) The consistency of the luminous efficiencies of lamps colour matched photo-electrically.

The first two of these tests have been discussed already.

With regard to the third criterion, experiments have been made and Table XII shows the photo-electric colour match errors as estimated by this method.

Table XII. *Errors in the Directional Photo-electric Colour Match Volts as estimated by Measurements with two Photo-electric Cells of different Sensitivity Curves.*

Lamp	Per cent. colour match voltage difference	Lamp	Per cent. colour match voltage difference
A	+ 0·21	*E*	− 0·35
B	− 0·22	*F*	+ 0·30
C	+ 0·37	*G*	+ 0·18
D	− 0·37	*H*	− 0·16

These results seem to indicate that different methods of colour matching lead to different results, but it must be remembered that the above results include the accumulated experimental errors of three measurements, so that these results by themselves cannot be regarded as conclusive evidence of an error.

With regard to the fourth test, the readings in Table X show the efficiency errors and these indicate that the photo-electric colour matches are in error by the amounts given in Table XIII.

When the results of these four different methods of checking the accuracy of the colour matching are considered and the results of the last row of Table IX, and Tables XII and XIII are compared, it seems to the author that the differences are rather greater than the experimental errors in the respective tests, and that consequently different methods of colour matching of electric lamps lead to slightly different results. This confirms the arguments already expressed and leads to the conclusion that it is extremely likely that differences of the colour of the

glass bulb prevent an accurate matching of frequency distribution and that with well-made lamps the resulting errors introduced may amount to as much as 0·1 per cent. voltage.

Table XIII. *Apparent Error in the Photo-electric Colour Match Voltages judged from the Observed Luminous Efficiency of the Lamps*

Lamp	Per cent. colour match voltage difference	Lamp	Per cent. colour match voltage difference
A	+ 0·54	*E*	+ 0·12
B	− 0·59	*F*	+ 0·50
C	+ 0·36	*G*	− 0·28
D	− 0·32	*H*	− 0·33

Thus, since for intensity measurements it is necessary to make a photo-electric cell act as a sort of mechanical eye of the same sensitivity curve as the normal eye, it is necessary also to colour match with apparatus which shows a similar response to colour differences as the normal eye. In the apparatus described above this might at any rate be partially effected by using two photo-electric cells, one more sensitive to red light than the eye and the other an equal amount more sensitive to blue. The difficulty in making such an arrangement is that if a rubidium cell were placed behind a filter so as to make it more sensitive to red than the eye, the total sensitivity of the cell-filter combination would be very small. With caesium cells this difficulty would be less, but it is believed that, with these, other considerable difficulties would be experienced. It is, further, by no means certain that such a combination would prove satisfactory if visual colour vision is trichromatic.

On the whole, therefore, it seems that in the colour matching of electric lamps a precision greater than 0·1 per cent. voltage has very little meaning. A similar conclusion has been reached by Campbell*, but on the view that even in the most uniformly drawn filaments the temperature along the filaments varies fairly considerably.

Discussion on Intensity Measurements

The intensity measurements do not appear to be as precise as the colour matching measurements, either in the cube or on the bench. The precision is only about two or three times that of the visual method, but it has in addition the advantage that it is entirely objective. The lamps, however, need to be previously colour matched and run at stated voltages unless a suitable filter is found for a rubidium cell to make it reproduce the sensitivity curve of the eye. With the results shown in this paper, errors in the intensity measurements due to faulty colour matching do not probably exceed 0·02 per cent.

Luminous intensity measurements made by the photo-electric method are very consistent, generally within 0·1 per cent. when made on the same day, but when

* N. R. Campbell, *loc. cit.*

a considerable time (say a month) elapses between measurements the results may differ by as much as 0·5 per cent. This phenomenon has been observed by the visual method and is more marked for some types of lamps than for others. For example, it is more marked for lamps *E*, *F*, *G*, and *H* than for lamps *A*, *B*, *C*, and *D*, by both photo-electric and visual methods. This is obviously more due to the lack of constancy in the lamps than to any fault of the method of measuring.

The question has been raised whether there is any difference between the photo-electric and visual results due to the fact that whilst in the photo-electric measurements the light falls on the cell aperture which is only about one inch in diameter, in the visual measurements the light falls on a diffuse reflecting surface which is over two inches in diameter. Thus the photo-electric method would tend to show up the peaks of the polar curves of the lamps more than the visual method and somewhat different results might be expected. This question has been investigated by moving the photo-electric cell which had a diffusing filter covering its aperture laterally one inch each side of its normal position, and it has been found that for the lamp used there is, for a two-inch displacement, a change in candle-power of $0{\cdot}45 \pm 0{\cdot}05$ per cent. For a vertical displacement the results were not so conclusive on account of the difficulty in setting the cell in its correct position, but here again there appeared to be a slight difference of illumination. One is led to the conclusion that on account of the fact that in the photo-electric method the light is more strictly unidirectional than in the visual, there may be, and is for some lamps, a discrepancy between the photo-electric and visual results of 0·1 or occasionally of as much as 0·2 per cent.

Another interesting fact is that, besides the slow and steady fatigue of the photo-electric cells which becomes almost negligible after an exposure of an hour or two to the illumination, there appear to be small fluctuations of sensitivity of the cells which limit the accuracy of the photo-electric method. Thus it is found that after a certain illumination on the cells has been reached, the accuracy of the results is independent of the illumination, and although the apparatus is of course much more sensitive when a 400-watt lamp is on the bench than when there is only a 30-watt lamp there, the results are just as consistent for the lower illumination as for the higher. Perhaps it is a little unfair to the photo-electric cells to blame them for this in view of the fact that there is no evidence to show that the lamps themselves are not responsible for it, but on the whole it appears more likely that the fault lies with the cells. In any case these fluctuations seem to limit the accuracy of luminous intensity measurements to about 0·1 per cent. and the only way to get rid of this is to use the same cell for both the comparison and test lamps and alternate fairly quickly the two beams of light. This raises new difficulties and sources of error and it remains to be seen from experiment whether these can be overcome.

Conclusions

For colour matching, the results are much more precise with photo-electric than with visual methods. The precision amounts to 0·1 per cent. voltage on electric lamps. On account of the colour of the glass bulbs of the lamps, however, it is not possible to obtain the same energy-frequency distribution curves for different lamps. Different methods of colour matching therefore lead to slightly different results. For well-made lamps, however, the difference between the photo-electric and visual colour matches does not in general exceed 0·1 per cent. voltage. No difference has been detected between the effect of rating well-made lamps by voltage and by current.

For intensity measurements, the photo-electric method is about two or three times as accurate as the visual. It is useful as a check on visual results, and is to be preferred where the highest accuracy is required.

In conclusion I should like to express my thanks to various members of the Photometry Division of the National Physical Laboratory for their assistance during this work, especially to Dr J. W. T. Walsh for his valuable advice and continued interest, and to Mr W. Barnett who superintended the visual results of this paper. My thanks are also due to Dr D. W. Dye for supplying the rolled Wollaston wire electrometer suspension.

DISCUSSION

Mr C. C. Paterson: It is very satisfactory to note the work which the N. P. L. has in hand on the application of photo-electric methods to photometry. The subject is a very large one when the measurement by colour match of filament temperature or efficiency is included in the programme. In the maintenance of photometric standards the more important technique to develop first is the direct comparison of light intensities, and one feels that the more complicated and less rigid "colour match" work could be left till later. I rather gather from Dr Harrison's paper that the N. P. L. will one day adopt photo-electric methods for their standard photometric work, and one would like to know whether they feel definitely that this will be so.

The author attributes differences between directional photo-electric and directional visual results for colour match to coloration introduced by the glass or presumably also by the film on its inside surface. He arrives at this result by inference, but I think he might have directly measured the effect and put the question beyond doubt by mounting the two filaments to be compared in the same bulb and using them alternately.

He has followed very closely some of the lines of work carried out by N. R. Campbell and with much the same conclusions. He started out, I think, to increase the precision of the measurements, but I gather that he found as Campbell did

that the limitation to accuracy lies not in the photo-electric methods but in the constancy attainable with even the best incandescent filament standard lamps.

Prof. L. C. Martin: Dr Harrison deduces from his observations that the spectral energy distribution of the sources compared in certain cases is not identical, and it appears that the visual comparison method employed the large angular field of the Lummer-Brodhun photometer head. Would it not be well in making visual colour matches to restrict the size of the visual field say to 2° in order to escape the uncertain retinal effects when comparing such sources in a large field? Visual colour matches call sometimes for a type of field different from that useful in intensity matches, but it may be in the present case that the differences of energy distribution are too small to affect the results.

Dr J. W. T. Walsh: I should like to express my appreciation of this paper. In following the work I have been struck by the large number of difficulties which beset the worker who is striving to apply photo-electric methods to photometric measurements, and I should like to congratulate Dr Harrison on the persevering way in which he has grappled with these difficulties one by one.

Perhaps he will permit me to answer Mr Paterson's question with regard to the adoption of this method of photometry at the National Physical Laboratory. Mr Paterson will, I know, be the first to appreciate the length of time which it takes to develop a technique in any branch of high precision measurement, and it is largely owing to his work at the N. P. L. that the technique of visual photometry has reached its present standard. To change over to a new technique without loss of precision will be a work of some considerable time, but I have little doubt that it will be undertaken during the course of the next two or three years.

Turning now to p. 204 of the paper, I should like to ask Dr Harrison whether he thinks it would be possible to substitute for the Wratten K 3 filter some chemical solution of definite and specified composition. If so, it appears to me that this would be more satisfactory than the use of a filter which is very liable to vary from one specimen to another.

Further, I should like to ask if he has considered the possibility of applying the photo-electric method in colorimetry. If it were possible to find a colour filter which, in combination with a suitable form of cell, would give a sensitivity curve closely approximating to that of one of the colour sensations of the eye, and if such a combination could be found for each sensation, it might be possible to simplify colorimetric work considerably.

Mr B. P. Dudding: On behalf of my colleagues and myself, I wish to say how pleased we are to know that work on photo-electric photometry is being pursued at the N. P. L. and we congratulate the author on the work he places before us. We do not, however, agree entirely with conclusions drawn by the author from the data presented. Referring to p. 204, "Discussion on Colour Matching," whilst agreeing that the most important point is the difference between the mean spherical

and directional colour matching we do not think that the author is entirely sound in stressing the difference between the photo-electric and visual results. For example, Table III shows the mean error of photo-electric mean spherical colour matching to be 0·11 per cent. Table IV shows the mean error of visual mean spherical colour matching to be 0·28 per cent., whilst the differences between the photo-electric and visual measurements given in Table V indicate a mean difference of 0·36 per cent. We submit that, considering only eight lamps were measured, this error is scarcely larger than would be expected from the errors in the foregoing Tables III and IV.

Considering in detail the difference between the directional and spherical colour matching, the author tabulates on p. 204 a number of points to which the difference may be attributed.

The different emissivity functions for different samples of tungsten and the different cooling effects at the ends of the filament have already been dealt with at some length in a paper by Campbell and Freeth read before the Optical Convention of 1926. With regard to this selective colour of glass; with this must be included the effect of the films of getter on the surface of the glass, which is likely to be the more important. In the same paper already referred to, Campbell shows that the colour of the glass affects the intensity measurement rather more than the colour matching.

Turning now to the difference between directional and mean spherical colour match, we think that not sufficient stress is laid on the possible effect of the cube and that it is a lack of uniformity of the cube, together with the differential blackening of the bulb, which are really the cause of the differences observed, rather than the selective absorption of the glass which is stressed by the author (see errors discussed on p. 208). If one refers to Table II it will be seen that a small movement of the filament introduces errors in colour matching of the same order of magnitude as the average difference being considered. This indicates strongly that the cube is not behaving in a uniform manner and that therefore differences are due to that rather than to differences between the glass of the various lamps. Further, the differences in intensity discussed on p. 208 indicate that some difference between mean spherical and directional colour match may be due to the small pencil of light that enters the photo-electric cell when making directional observations.

We wish to emphasise the paragraph on p. 207 pointing out that a precision greater than 0·1 in voltage is not likely to be obtained, owing to lack of uniformity in the filament itself. With regard to the use of cells having widely different sensitivity curves, Campbell and New (*Journ. Scient. Instr.* **3** 1925–6 (2)) have already shown that the differences in colour match determination are quite small for considerable changes in the sensitivity curves of the cells employed.

Lastly, the ultimate limit to the sensitivity of the photo-electric method, which the author discusses in his final paragraph, is almost certainly due, not to irregularities in the behaviour of the cells, but to irregularities in the behaviour of the light. The action of light on a photo-electric cell (and probably any receiving

instrument, including the eye) is fundamentally discontinuous and in part fortuitous. No experimental method can possibly avoid irregularity due to this cause. The "small fluctuations in sensitivity" which the author mentions are nothing but "Schweidler fluctuations" in the mean intensity of the light measured over a finite period.

Mr T. Smith: What precautions were taken in the tests to eliminate irregularities that might be caused by the K filters employed?

Dr Harrison: I should like to thank those who have contributed to the discussion of this paper for the interest that they have taken in it and for their valuable remarks upon the subject. I agree with Mr Paterson that experiments with double filament lamps would be interesting, although even in this case differential colouring of the bulb would not be entirely eliminated. It would also be necessary to take precautions against thermionic interactions between the two filaments. There would be no difficulty in carrying out such experiments but the lamps would have to be specially made. In this, perhaps, Mr Paterson might be able to help us.

I used an ordinary Lummer-Brodhun photometer head for the visual colour matching and I do not know whether a smaller angular field would lead to different results.

With regard to a suitable filter for use with a rubidium cell in order to give an approximation to the colour sensitivity of the eye, I have found that a fairly concentrated solution of naphthol yellow in distilled water, contained in a glass absorption cell 1 cm. thick, gives perfect results over a range of colour corresponding to a 10 per cent. change in voltage on a lamp run at normal efficiency. Using photo-electric cells for colorimetry in the manner suggested by Dr Walsh is a distinct possibility, although much work would have to be done before reliable results could be obtained.

Mr Dudding's remarks are extremely valuable in calling attention to the salient features of this paper, and show clearly that too much care cannot be expended in examining data before drawing any conclusions. I am afraid, however, that, although I have been very interested in, and impressed with, his critical examination of some of the figures, he has not quite converted me to his faith in the identity of the photo-electric and visual colour matching results. For example, if the error of photo-electric colour matching is 0·11 per cent. and that of visual colour matching is 0·28 per cent., then the average difference between the photo-electric and visual results should be about 0·28 per cent., which is less than 0·36 per cent. In the same way, for the directional colour matching, the average difference between the photo-electric and visual results is 0·31 per cent., which is greater than 0·25 per cent., the experimental error estimated in the manner indicated above. It is also to be noted that the average difference between the photo-electric and visual colour matching is considerably less than the difference for certain outstanding lamps.

I agree that the differential blackening of lamps combined with a lack of

uniformity of the cube is an important factor, but I doubt whether this can be held wholly responsible for apparent differences between spherical and visual colour.

With regard to the precautions with filters, it is fortunate that the same filter could be used all the time and in the same manner throughout a complete set of readings so that any irregularities were eliminated. Had this not been possible, a great deal of difficult calibration would have been necessary.

ON BREWSTER'S BANDS. PART II

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[Communicated by Prof. C. V. Raman, F.R.S.]

MS. received, 26th March, 1927. Read, 9th June, 1927.

ABSTRACT. The paper is a continuation of the one published in these *Transactions* under the same title. It discusses the nature of the patterns obtained by the superposition of two systems of Haidinger's rings when the actual law of spacing of the rings is taken into consideration. Figures are reproduced illustrating the patterns, and a theoretical deduction of these is given.

1. INTRODUCTION

IN Part I* of this paper, published jointly with Prof. C. V. Raman, the explanation of the colours of thick plates was discussed from the new viewpoint suggested by Schuster†, and illustrations were reproduced showing how, when the Haidinger ring-systems due to two plane-parallel plates are superposed on each other, in the sense of their reflecting and transmitting powers being *multiplied*, differential and summational systems arise, the former corresponding to the well-known Brewster's bands observable in white light. In the discussion it was assumed that the Haidinger rings in each plate are spaced in the manner of a zone plate, that is according to the law of the square roots of the natural numbers. It is known, however, that this is true only for moderately small angles of incidence, and even then only approximately. Actually, the Haidinger rings in a glass plate are spaced somewhat differently, the successive rings becoming closer and closer together only up to about an angle of emergence of 45° and then widening out again. If i be the angle of emergence from the plate, the law of spacing of the Haidinger rings in a parallel plate of refractive index μ and thickness t may be written in the form

$$2t\sqrt{\mu^2 - \sin^2 i} = n\lambda \qquad \ldots\ldots(1).$$

In the present paper it is proposed to discuss the nature of the patterns obtained by the superposition of two similar systems of Haidinger's rings when the actual law of spacing of the rings is taken into consideration. The case of two plates of unequal thickness will also be considered.

* C. V. Raman and S. K. Datta, *Trans. Opt. Soc.* **27** 1926 (51).
† A. Schuster, *Phil. Mag.* **48** 1924 (609).

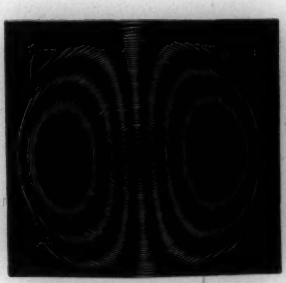

Fig. 1

Fig. 2

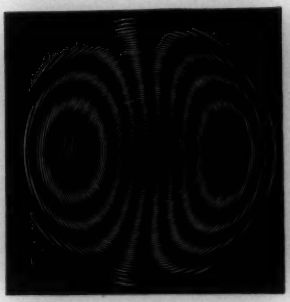

Fig. 3

Fig. 4

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2. Effect of Superposing Two Haidinger Ring-Systems

In order to determine the actual spacing of the Haidinger rings, the angles of emergence for different rings were calculated by substituting for n successive integers in the equation (1) for a plate of known thickness and for light of known wave-length, μ being the refractive index of the material of the plate for the light of that particular wave-length. Taking $i = 0$ as the centre and the radii of the successive rings equal to the respective angles of emergence, the outermost one being $i = 90°$, a series of circles was engraved with a milling machine on a nicely faced copper plate. The spacing of the rings on the plate thus corresponds to that of an actual Haidinger ring-system. To illustrate the effect of multiplying the distributions of light intensity in two sets of Haidinger rings when their centres are not coincident, the prints reproduced in Figs. 1 to 4 were prepared. Figs. 1 and 2 were obtained by printing the same copper plate twice on the same paper after displacement, thus representing the case of plates of equal thickness, and Figs. 3 and 4 by printing from two copper plates having rings of two different sizes, thus corresponding to the case of plates of unequal thickness. In Figs. 1 and 3 this distance between the centres of the superposed ring-systems is small, while in Figs. 2 and 4 it is greater. The characteristic feature shown in the figures is the double system of approximately elliptic bands, whose major axes run transverse to the line joining the centres of the ring-systems. Figs. 1 and 3 give the form of Brewster's bands for two plates inclined to each other at a small angle, the bands being very broad in this case, but coming closer and closer together with increasing inclination of the plates. In Figs. 1 and 2 the central band is approximately straight and the pattern is symmetrical. In Figs. 3 and 4 the central band shows a marked curvature, and the patterns are clearly unsymmetrical, the ellipticity of the bands on one side being greater than on the other side of the central band. The initial narrowing and subsequent widening of the Haidinger rings, as we proceed from the centre towards the margin of each figure, can be clearly seen in the illustrations.

3. Approximate Theory

We have now to consider the manner in which the superposition patterns are formed. Taking the most general case in which the two sets of rings are of different size, we may, following Schuster*, take the centre of each system to be a point of maximum intensity—the radii of the rings having maximum intensity being no longer in the ratio of the square roots of the successive integral numbers, which is only valid when i is small. Considering the two plates to be of thicknesses t_1 and t_2 but of the same refractive indices, the law of spacing of the two systems of rings is given by the equations

$$\left.\begin{aligned} 2t_1\sqrt{\mu^2 - \sin^2 i_1} &= n_1\lambda \\ \text{and}\quad 2t_2\sqrt{\mu^2 - \sin^2 i_2} &= n_2\lambda \end{aligned}\right\} \qquad \text{.....(2).}$$

* A. Schuster, *Phil. Mag.*, *loc. cit.*

Referred to a system of coordinates in which the line joining the two centres is the axis of x and the origin is a point midway between the centres, the equation (2) for the rings becomes

$$\left.\begin{aligned} 2t_1\sqrt{\mu^2-\{y^2+(x+a)^2\}/R_1^2} &= n_1\lambda \\ \text{and}\quad 2t_2\sqrt{\mu^2-\{y^2+(x-a)^2\}/R_2^2} &= n_2\lambda \end{aligned}\right\} \quad \ldots\ldots(3),$$

where n_1 and n_2 are integers and R_1 and R_2 are linear constants. From these equations by addition and subtraction we obtain

$$t_1\sqrt{\mu^2-\{y^2+(x+a)^2\}/R_1^2} \pm t_2\sqrt{\mu^2-\{y^2+(x-a)^2\}/R_2^2} = (n_1 \pm n_2)\,\lambda/2$$

as the equation for the summational and the differential systems respectively, according as we use the positive or the negative sign; after squaring twice on both sides and appropriate reduction this transforms into the equation

$$\alpha^2(x^2+y^2)^2 + Ay^2 + Bx^2 - 4Dx + 4F(y^2x + x^3) + C = 0 \quad \ldots\ldots(4),$$

where

$$\alpha = \frac{t_1^2}{R_1^2} - \frac{t_2^2}{R_2^2},$$

$$\beta = \frac{t_1^2}{R_1^2} + \frac{t_2^2}{R_2^2},$$

$$\gamma = \frac{(n_1 \pm n_2)^2\lambda^2}{2},$$

$$A = 2a\{a^2\alpha - \mu^2(t_1^2 - t_2^2)\} + \gamma^2,$$

$$B = A + 4a^2\beta^2,$$

$$D = a\{\mu^2(t_1^2 - t_2^2)\beta - a^2\alpha\beta - \alpha\gamma/2\},$$

$$F = a\alpha\beta,$$

and

$$C = \{a^2\alpha - \mu^2(t_1^2 - t_2^2)\}^2 + \gamma\{\gamma/4 + a^2\beta - \mu^2(t_1^2 + t_2^2)\}.$$

If we assume that the difference in thickness between the plates is small we can neglect the terms multiplied by $(t_1^2 - t_2^2)$ and α in comparison with other terms. The equation (4) then reduces to

$$A'y^2 + B'x^2 + C' = 0 \quad \ldots\ldots(5),$$

where A', B', and C' have the values obtained by putting $\alpha = 0$ and $t_1^2 - t_2^2 = 0$ in the values of A, B, and C. The superposition curves represent therefore a system of ellipses with semi-axes $\sqrt{C'/A'}$ and $\sqrt{C'/B'}$. These can never degenerate into circles because the ratio of the semi-axes, viz. $\sqrt{B'/A'}$, i.e. $\sqrt{1 + 4a^2\beta/\gamma}$, can never become equal to unity, a conclusion also reached by Blasius* by a different method.

In the case when the two plates are of exactly equal thickness we obtain for the superposition curves the equation

$$A_1y^2 + B_1x^2 + C_1 = 0 \quad \ldots\ldots(6)$$

by putting $t_1 = t_2$ in the equation (5) above, where the values of A_1, B_1, and C_1

* E. Blasius, *Wied. Ann.* **45** 1892 (333).

are obtained from those of A', B', and C' by putting $t_1 = t_2$, an equation analogous to that obtained by Lummer* by a different method. The equation (6) above also represents a set of confocal ellipses, whose two semi-axes are given by $\sqrt{C_1/A_1}$ and $\sqrt{C_1/B_1}$. Since $B_1 > A_1$, $\sqrt{C_1/A_1} > \sqrt{C_1/B_1}$; the major axes of the ellipses are thus parallel to the y-axis, that is, transverse to the line joining the centres of the ring-system.

From the equation of the summationals

$$2t\,[\sqrt{\mu^2 - \{y^2 + (x+a)^2\}/R^2} + \sqrt{\mu^2 - \{y^2 + (x-a)^2\}/R^2}] = (n_1 + n_2)\,\lambda$$

we can infer that the left-hand side can never become equal to zero, while in the case of the differentials

$$2t\,[\sqrt{\mu^2 - \{y^2 + (x+a)^2\}/R^2} - \sqrt{\mu^2 - \{y^2 + (x-a)^2\}/R^2}] = (n_1 - n_2)\,\lambda$$

it is possible to find the differential curves corresponding to $n_1 - n_2 = 0$. When $n_1 - n_2 = 0$ the equation (6) of the second degree transforms to one of the first degree. In this case equation (6) represents a straight line passing through the origin and parallel to the axis of y.

4. More Exact Theory

The equation (6) deduced in the previous section may be considered only as an approximation carried one step further than those given in Part I of this paper. In reality the Haidinger rings are described on the surface of a sphere and not on a plane surface. The form of the superposition curves on the entire spherical surface was deduced by Ketteler†, Lummer‡, Blasius§, and others. According to Lummer the form of the superposition curves on the entire surface of the sphere is given by

$$\sin^4\rho \sin^2 2\phi \cos\delta - \sin^2\rho\,\{k^2\cos^2\phi + (4\cos^2\phi - k^2)\cos^2\delta\sin^2\phi\} + k^2\left(n^2 - \sin^2\phi - \frac{k^2}{4}\right) = 0,$$

where 2ϕ is the angle between the plate normals, ρ the angle between the incident ray and the line bisecting the angle between the plate normals, δ the surface angle, n the refractive index, and k equals $\dfrac{(n_1 \pm n_2)\,\lambda}{2t}$ according as we consider the above equation as the equation of the summationals or of the differentials. Thus the superposition patterns are not ellipses but curves represented by the above equation of Lummer. This departure from the elliptic shape is well exhibited in Figs. 1 and 2.

The investigations were carried out in the laboratory of the Indian Association for the Cultivation of Science at the suggestion of Prof. C. V. Raman, F.R.S., and the author is indebted to him for the unfailing interest he has taken during the progress of the work.

* O. Lummer, *Wied. Ann.* **24** 1885 (429).

† E. Ketteler, *Beobachtungen über Farbenzerstreuung der Gase*, Bonn, 1865; also Müller-Pouillet, *Lehrbuch der Physik*, **2** (Optik) 1926 (712).

‡ O. Lummer, *Wied. Ann.* **24** 1885 (417).

§ E. Blasius, *Wied. Ann.* **45** 1892 (316).

REVIEWS

The Pictorial Annual of the Royal Photographic Society, 1926, and a Critical Dissertation. By F. C. TILNEY, F.R.P.S. Pp. iv + 52; 53 Illustrations. (London: Simpkin, Marshall, Hamilton, Kent and Co., Ltd.) Price 5*s.* net; bound in cloth, 7*s.* net.

For this new Annual, dealing with the pictorial side of photography, we have to thank Mr Tilney. He has not only selected the pictures here reproduced, and written the text, but the suggestion, happily welcomed by the Royal Photographic Society, that a book dealing with the photographs of the year should be published, came from him. The present volume is the first of what we hope will become an established undertaking by the Society.

Mr Tilney has great faith in the social influence of the camera, and would help others to see what delights him in the pictures he has chosen. That he seeks to cover the whole pictorial field will be seen from the titles of his chapters. In addition to an Introduction and a final discourse entitled The Camera: a Cultural Influence, he writes on Landscape: Shore and Lake Scenes: Architecture and Street Scenes: Portraits and Figure Studies: and Still Life. Each of these groups is aptly illustrated by photographs, of which the volume contains fifty-three. The reproductions are excellent, despite some inevitable loss in quality in comparison with the original prints. The "dissertation" is fresh and pleasing, and the book should be read by all who are interested—and who is not?—in photography. There is even hope that those who regard the camera as a mere record maker—who, in our author's words, dub anything falling short of an exhaustive inventory of minutiae poor photography—may come to understand that the camera can produce good pictures which are also good photographs, if only they will read this charmingly written and well-printed book of Mr Tilney's. They may even go farther, and learn to produce them!

Stars and Atoms. By Prof. A. S. EDDINGTON, M.A., D.Sc., LL.D., F.R.S. Pp. 127; 11 Figures. (Oxford: Clarendon Press, 1927.) Price 7*s.* 6*d.*

This book is based on an Evening Discourse given at the meeting of the British Association in Oxford in August 1926 and a course of three lectures delivered at King's College, London. It is written in the form of three lectures entitled "The Interior of a Star," "Some Recent Investigations," and "The Age of the Stars," and may be looked upon as a popular account of the main ideas and results contained in the author's larger book *The Internal Constitution of the Stars* (Cambridge University Press, 1926). Probably no one is more competent than Prof. Eddington to talk about the many recent developments in astronomy which have followed in the wake of the great advances made in atomic physics. As might be expected from the title of this book, the subject is a most fascinating one and it is treated in a very charming manner. The author makes use of quite striking similes and metaphors which help the reader to grasp more easily the points brought out. For example, in referring to the stream of X-rays in the stars, he says that "It is like an army of mice marching through your larder springing the mouse-traps as fast as you can set them." In another place he says "Stellar atoms are nude savages innocent of the class distinctions of our fully arrayed terrestrial atoms." To quote one more example, in referring to nebulium the author says that "It is some quite familiar element which we cannot identify because it has lost several of its electrons. An atom which has lost an electron is like a friend who has shaved off his moustache; his old acquaintances do not recognise him. We shall recognise nebulium some day." The reader who can lay down such a book as this before he has read every word of it must indeed suffer from a lack of love for romance.

X-Rays Past and Present. By V. E. PULLIN and W. J. WILTSHIRE. Pp. 229; 43 Figures. (London: Ernest Benn, Ltd., 1927.) Price 12*s.* 6*d.* net.

In the preface the authors say "We have set out to tell the story of X-rays for the benefit of the interested and enquiring layman because we believe that no general account of this fascinating branch of science is so far available for him." If, after perusal of this book, the "interested and enquiring layman" is not more than satisfied, the fault must lie with him and not with the authors, for they have succeeded in producing a most interesting account of this important subject. It is obvious that the authors possess not only a first-hand knowledge of X-ray technique but also the power of explaining scientific problems in a very simple and clear manner. To the scientific reader the account of the authors' own work in the Research Department, Woolwich, will probably be found the most interesting part of the book. The photographs illustrating their results are excellent. The book is practically free from mistakes; the only ones noticed are that "Röntgen and Römer are spelt throughout the text with unaccented o's and that von is spelt with a capital v, but these are quite minor points. The book, although primarily written for the layman, can be recommended to scientific workers who should find it a useful source of information on the most recent applications of X-rays in medicine, surgery, and industry.

ABSTRACTS OF PATENT SPECIFICATIONS

256357. Barr and Stroud, Ltd., and another. SPOTTING APPARATUS.

In the field of view of the sighting-telescope of an apparatus for spotting gun-fire there is arranged a series of vertical locating lines readily distinguishable from each other by their character, and a movable index which can be set upon the splash of the shot. The necessary correction can be transmitted to the gun in terms of angular deflection. In adapting the instrument for range correction, the reference lines are placed horizontally, and the correction applied is proportional to the square of the range.

256449. Heurung, A. OPTICAL SYSTEMS.

A pair of images, turned through a right angle with respect to the original, is produced by mounting in front of the objective a prism in which three reflections take place in one plane at 45° to the horizontal. The entrant and emergent faces are preferably perpendicular to the axis. From the initial image so obtained, two images side by side are formed upon a film by a pair of auxiliary lenses.

256473. Delaux, P., and others. VEHICLE LAMPS.

The bulb is backed by a concave reflector, and has in front of it a convex lens of which the lower part is cut away to form a prism to direct the light downwardly. A hemispherical reflector with a central aperture may be movable about the bulb, so as to be capable of taking up a position between it and either the lens or the main reflector.

256586*. Zeiss, C. LENSES.

Numerical data are provided for the construction of lenses free from chromatic and spherical aberrations, astigmatism, and coma. The system comprises a dispersive member between two collective members, one of which is a cemented triplet.

* Specification published before acceptance.

256611*. Tawil, E. P. OPTICAL SYSTEMS.

Various optical arrangements as follows are employed in connection with a television apparatus in which variations of light intensity are controlled by means of a piezo-electric crystal. In one instance the image of an illuminated slot, comparable in size with the variations in length produced in a quartz plate, is formed upon the edge of the plate, so that it intercepts the light in varying degrees. The light passing, or that which is intercepted and reflected from an element upon the plate, is concentrated by a lens at a convenient point. In a modification the arrangement is such that the plate is bent under the influence of the electrical changes, and a varying divergence thus imposed upon the beam reflected from a silvered surface of the plate. An interposed perforated screen passes a varying amount of light, which is collected as before. Applications of Fresnel mirrors, double refraction, and polarisation are also referred to.

256776. Gandell, W. R. K. OBSERVATION DEVICES.

To enable the driver of a vehicle to obtain an early view of possible traffic at a cross road, a system of mirrors is so mounted that no lateral inversion shall arise. The objective element consists of a convex mirror or a concave lens backed by a plane reflector. A similar system may be used upon an aeroplane to obtain a view above and below, or the device may be fixed in position at cross roads.

256879. Steinberg, E. PROJECTORS.

A light projector has arc electrodes placed parallel to the optical axis, an adjustment of the reflector or of the electrodes being provided to permit of the correction of the focus as the electrodes are consumed.

257046. Eden, J. L. STEREOSCOPY.

In order to obtain a stereoscopic or relief effect in cameras or telescopes, a fitting is employed consisting of a plano-convex lens and a plano-concave lens of the same power, a plane-parallel glass plate or a pair of opposed prisms being placed between them, covering half the field. The effect produced is that of displacing laterally a portion of the light from each point of the object.

257546. Romer, J. F. OPTICAL SYSTEMS.

This and the three following specifications describe optical systems designed to produce a number of images alike as to point of view, size, and outline. Colour screens interposed in the paths of the beams enable the images to be used in colour photography, and the same systems serve to combine multiple colour images for optical projection. In the first system the beam emerging from an objective is refracted from the inclined face of a prism on to a semi-transparent surface common to this prism and to a second similar prism adjoining it. The divided beam emerging is again refracted, and produces two similar but reversed images side by side. Each of the emergent beams may be again split in a similar manner, thereby producing four images, lying either in a line, or in a square formation.

257547.

In this case light from the object impinges first upon a double prism which splits it into two as above, and it is then received upon a compound prism having within it an oblique semi-transparent surface. That part of the light which is reflected at this surface is again reflected at an oblique face of the prism, the remainder passes on. Four lenses receive the emergent beams, forming erect images in square formation, arranged as back-to-back pairs.

* Specification published before acceptance.

257548.

With this system four erect images facing the same way and arranged in square formation are obtained. The light is directly incident upon one face of a compound prism, and the beam is divided at a semi-reflecting surface, one portion passing on, the other being reflected at an external surface of the prism. Both beams then pass to a second similar prism, lying in a cross plane. Separate lenses form the four images.

257549.

This is a fourth system of the same general type, but in this case the light reflected from the semi-reflecting surface is again reflected normally from a fully silvered face of the prism. Separate lenses produce a pair of laterally reversed images.

257920*. Poulsen, A., and another. OPTICAL PHONOGRAPHS.

A sound record produced by photographic means is projected through a system, such as a pair of crossed cylindrical lenses, designed to distort the image unequally in two directions, so that it shall conform to the shape of a photo-electric cell in the reproducing apparatus, or to that of a slit or aperture placed in front of it.

258092. Warmisham, A., and another. LENSES.

The zonal spherical aberration of objectives of the Petzval type is reduced, permitting an increase in working aperture, by dividing the front convergent member, or the convergent component of the back member, into two convergent portions.

258215*. Bosch Akt.-Ges. VEHICLE LAMPS.

A portion of the front glass is provided with vertical hemi-cylindrical ribs to furnish lateral illumination, while upon the remainder is formed a series of vertical ribs merging at the lower half into concentric semicircles.

258482. Stephens, A. J. LAMP FRONTS.

The front glass of a vehicle, street, or other lamp adapted to illuminate a long rectangular area consists of a dioptric lens covered by a glass plate upon which light-directing prisms are formed. These may consist of a vertical series on one face, and a single or double group of inclined lattices on the other. A number of different arrangements are described.

258755. Parodi, R. PERISCOPES.

Two similar annular toric lenses symmetrically placed at the ends of the tube are used in connection with an intervening axial system to project upon a cylindrical screen an image extending round the horizon. Part of the light may be intercepted for direct observation. An image of the zenith region is formed by axial lenses in conjunction with the same intermediate lenses.

259341. Cubitt, A. S. STEREOSCOPY.

In order to obtain a relief effect, a camera is fitted with a lens of large aperture, and a diaphragm with a small aperture is moved across or around the centre of the lens. For kinematography, the diaphragm is geared with the shutter so that exposures are made from opposite sides of the lens.

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259501. Redfern, C. G. LENSES.

To remove the astigmatism of the reading portion of a fused bifocal lens, the entire blank is heated upon a block of toric form, upon which the lens settles. The curvatures are suitably chosen to correct the astigmatism of the spherical segment.

259502. Redfern, C. G. LENSES.

The astigmatism inherent in a fused bifocal lens is counteracted by grinding an optical surface upon the reading lens side of the main lens with sufficient astigmatic power to provide the necessary correction.

259691. Taylor, W., and others. LENS MOUNTS.

To avoid distortion in mounting, the lens is held between two or three sets of three isolated abutments formed upon the mount, the lens, or a separate fixing piece.

260142. Dunhill, A. COLOUR EFFECTS.

Superimposed discs having coloured sectors are rotated at different speeds in front of a lamp, and the light emerging through an aperture in the casing, diffused by a glass hemisphere or prismatic lens, is thrown upon a translucent globe which forms the illuminated object.

260578*. Bradford, J. R. PROJECTION APPARATUS.

A film is mounted upon a transparent carrier-drum, through which the light passes to a projection lens and a deflecting prism mounted upon the spindle of the drum. The rotation of the drum may control the switch.

260801. Warmisham, A., and another. LENSES.

The two halves of the lens are separated by an air-space, and each half consists of two cemented doublets separated by an air-space. Each doublet includes a convergent and a divergent element, and is meniscus in form. The doublets adjacent to the diaphragm are convergent, the others being divergent.

260976*. Cushing, G. H. VEHICLE LAMPS.

The bulb is backed by a parabolic reflector continued into a cylindrical extension. The light then passes to a secondary reflector built up from a series of plane or merging sections, to produce the required distribution of light. The whole may be enclosed in a casing having a visor-shaped hood.

260982*. Sisson, H. W. VEHICLE LAMPS.

A reflecting element in the form of a cone or pyramid with its apex directed towards the light is fixed upon the inner side of the front glass, thereby obstructing the central portion of the direct beam, and diffusing the light laterally.

261028*. Bourdereau, A. PROJECTION APPARATUS.

Three separated lenses or equivalent systems, placed between the source of light and the gate, are so designed as to throw an erect image of the source into the projection lens, and an image of the entrance pupil of the system upon the gate.

261065. Ionides, A. C., and another. SIGHTING-DEVICES.

An inverted and reversed image of the object, formed by two pairs of inclined reflecting surfaces, is seen projected upon the object itself, the common point indicating the

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direction of aim. The reflectors are preferably formed by the faces of a compound prism, which can be tilted to provide a range-setting adjustment. The system may be rendered water-tight by cementing the elements in their cells by means of a low-melting alloy.

261195. Belin, E. TELEVISION APPARATUS.

A real optical image is explored by two mirrors, mechanically oscillated about axes at right angles, one at a high speed, the other at a low speed. The reflected ray, of varying intensity, is thrown upon a photo-sensitive cell in the transmission system. A synchronised system at the receiving end reconstitutes the image.

261326. Rudolph, P. LENSES.

Data are furnished for the computation of lenses corrected for coma, astigmatism, and all colour aberrations of the first order, and for spherical aberration of the second order. The lens consists of four or five single or cemented lenses alternately collective and dispersive, separated by air-gaps.

261354*. Bleyer, O. REFLECTORS.

A reflecting surface is formed upon glass by heating the article, and then immersing it in a bath of a molten tin-lead alloy, which adheres to the glass.

262020. Kondo, K. DIFFRACTION GRATINGS.

A plate coated with bichromated gelatine is exposed under an original grating, developed in hot water, and the surface silvered. For violet-end work, it may be further coated with nickel. Curved reflecting gratings may be similarly prepared.

262094*. Zeiss, C. OPTICAL APPARATUS.

In optical sighting-apparatus as described in Patent No. 240426, an additional mark is placed upon the surface of the collimator lens to enable the direction of its axis to be determined. By suitable graticules and scales, the distance between the axis of the collimator and the telescope can be ascertained.

262188. Henderson, J. B. SIGHTING APPARATUS.

In a sighting-device for use upon a moving platform such as a ship, a gyroscope is incorporated to rock a glass plate through an angle proportional to the velocity of roll, and thereby tilt the line of sight. In a modification, a thin prism is moved along the line of collimation of the telescope.

262412*. Zeiss, C. OPTICAL APPARATUS.

The apparatus is employed to test the alignment of the axis of a shaft with respect to two parallel axes, such as the axes of engine cylinders. A telescope is first adjusted with respect to the axis of one cylinder, and sighted upon marks carried by reflecting prisms on a tube supported within the shaft bearings. The tube having been adjusted, the parallelism of the axis of the second cylinder can be tested.

263009. Kirby, Inc. LIGHTING.

In order to produce substantially uniform illumination upon an object, an electric lamp having its filament in one plane is associated with a curved reflector so placed behind the lamp as to provide a converging beam, and with a lens or prism plate placed in front of the lamp, and sufficiently near to it to produce a divergent beam. The arrangement is such that the convergent and divergent beams intersect at the plane of the object.

* Specification published before acceptance.

263180*. De Forest Phonofilms, Ltd. OPTICAL APPARATUS.

In a device for recording or reproducing sound by optical means, three opaque bodies with straight edges are placed between the source of light and the film, in place of the usual slit. One of the bodies may be adjustable, in order to permit only a thin band of light to pass.

263414. Macnab, J., and another. REFLECTORS.

A rear reflector for cycles consists of a roughly frusto-conical glass body of which one end forms a convex lens element, the other being in contact with a concave reflecting surface struck from the same centre. Light incident upon the device over a considerable range of angular direction is returned along its own path.

263742*. Garbarini, A. REFLECTORS.

Auto-collimating reflectors, which may be used upon aerodromes, or fitted to buoys, consist of two hemispherical elements of different radius, one being preferably coloured, the other silvered. The smaller element may consist of rather more, or rather less than a hemisphere, to render the element visible at a predetermined distance.

263924. Tintometer, Ltd., and another. COLOUR APPARATUS.

In estimating the colour of a transparent or translucent substance, the apparatus is directed towards a tray arranged at an angle of 45°, and carrying a sheet of glass of known thickness and tint covering a white powder, the whole being illuminated by a known lamp directed at an angle of 15°. In this way, direct light is prevented from reaching the observer.

264039. Wade, H. PHOTOMETERS.

In photometers employing a standard lamp to produce a graded illumination upon a translucent screen, a resistance having a positive temperature coefficient is placed in circuit with the lamp so that the current is kept as uniform as possible while the battery is working out.

264174*. Telefunken Ges. TELEVISION.

A beam of light is passed by an exploring mirror through an object such as a kinematograph film, and the transmitted portion falls upon a sensitive cell. The resulting varying currents, after amplification, are applied to the plates of a Kerr cell to produce variations in intensity in a beam which is projected to the receiving station. A similar apparatus is used to reconstitute the image. Alternatively, the Faraday effect may be utilised instead of the Kerr effect.

264191*. Granichstädten, R. PROJECTION APPARATUS.

Pictures for opaque projection are laterally inverted, so that when placed in the projector upside down they are correctly thrown upon the screen without the aid of a mirror. Illumination is effected by lamps and a hemispherical mirror, through which the objective tube passes.

264615. Smith, K. R. SIGHT EXERCISERS.

An appliance for developing binocular vision consists of a frame fitted with screens so arranged that one portion of the object is seen with both eyes, but other portions are seen with one eye only.

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* Specification published before acceptance.

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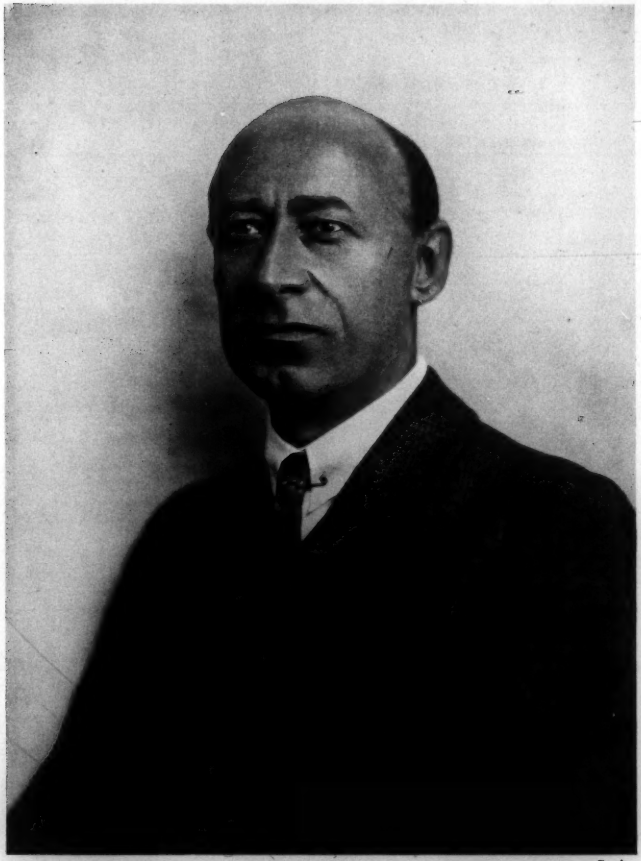

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